Loan Rate Deregulation and Credit Market Signalling Equilibrium

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In many developing nations, one of the regulations being phased out is the requirement that banks apply a common loan rate to all borrowers. Abolishing such a requirement allows banks to charge risk-adjusted loan rates based on borrowers' credit qualities. To better understand the economic consequences of loan rate deregulation, this paper analyzes its effects on aggregate credit supply and social welfare. We show that in the full information scenario when banks fully observe individual borrowers' credit qualities, both aggregate credit supply and social welfare increase with the deregulation. In the asymmetric information scenario when banks do not observe them, on the other hand, aggregate credit supply is likely to increase but the effect on social welfare is in general ambiguous. The reason why aggregate credit supply is likely to increase is because, in order to credibly signal their true credit qualities to banks, higher credit quality borrowers demand more than what they'd have demanded at the common loan rate. Due to this over-investment possibility, social welfare could decrease. (*JEL* Classification: D82, E51, G28)

I. Introduction

Korea's credit markets had been rather heavily regulated until

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the early 1990s when the government began expediting financial liberalization and deregulation. One of regulations having been phased out since then was the requirement that banks apply a common loan rate to all borrowers. Contrary to the competitive credit market where loan rates do reflect borrowers' creditworthiness, the regulator used to require banks to charge a common loan rate to all borrowers irrespective of their credit qualities.[1] However, this requirement has been abolished since January 1996.[2]

An adverse consequence of interest rate deregulation claimed by many is that it encourages banks to increase credit supply so that the aggregate credit risks borne by the banking industry increase. Examples might include the US credit markets in the early 1980s, Japan's credit markets starting from the late 1980s, and the Scandinavian experiences in the mid 1980s. Although this claim is intuitively appealing, whether an increase in banks' credit supply, if any, was due to interest rate deregulation, concurrent economic expansion, or any other reasons remains largely unanswered. Furthermore, whether interest rate deregulation and the potential increase in banks' credit supply associated therewith are welfare improving or not has yet been meaningfully addressed in the literature.

The purpose of this paper is to analyze the effects of loan rate deregulation on banks' credit supply (or firms' aggregate investments) as well as on social welfare. In the regulated loan rate regime, the regulator fixes a loan rate which banks are required to apply to all their borrowers. In the deregulated loan rate regime, on the other hand, it allows banks to determine their own loan rates for different borrowers which would become fully risk-adjusted through competition. To analyze and compare the said economic consequences of loan rate deregulation, we consider two informational scenarios; i.e. the full information scenario when banks fully observe firms' credit qualities and the asymmetric information scenario when they do not.

[1]In this paper, the regulator is meant to include all government agencies and institutions that have legal and administrative powers to control and guide banks' lending policies.

[2]Banks in Korea had been allowed to apply only two different loan rates, i.e. higher one for large corporations and lower one for small- and medium-sized corporations. In January 1996, the Bank of Korea abolished this restriction and, in principle, allowed loan rates to be risk-adjusted.

In the regulated loan rate regime with full information, banks can make use of credit quality information only for their borrower selection. That is, banks may have loan demands of qualified firms (with whom loans are profitable at the common loan rate) satisfied but reject those of unqualified firms. In equilibrium therefore banks earn some positive profits, while borrowers tend to under-invest, i.e. invest less than what is socially efficient.

With incomplete information, on the other hand, banks employ loan granting probabilities as a selection device. Here better quality borrowers would increase investments (or loan demands) in order to credibly signal their true credit qualities and hence to raise their loan granting probabilities. In this situation, a signalling equilibrium is shown to exist where all the qualified firms demand no less than a hurdle amount which is just enough to discourage mimicking behavior by the unqualified. The reason why investment serves as a signal is because a firm's marginal profit associated with an increase in investment increases with its credit quality.[3] In this equilibrium banks also earn some positive profits at the common loan rate by lending only to the qualified firms. This equilibrium is dissipative because most borrowers' investment levels turn out to be socially inefficient. In each informational scenario, the regulator sets a common loan rate to maximize social welfare.

In the deregulated loan rate regime, competition among banks gives rise to risk-adjusted loan rates. At these loan rates all loan demands are fulfilled, and banks break even with each and every borrower. In the full information scenario, on one hand social welfare will be at its theoretical maximum as the risk-adjusted loan rates dictate the socially efficient investment level for each and every borrower. In the asymmetric information scenario, on the other hand, an informationally consistent signalling equilibrium in the sense of Riley (1979) is shown to exist. This signalling equilibrium is also dissipative as borrowers tend to over-invest for the same reason as mentioned above. That is, firms with successively higher credit qualities engage in successively higher

[3]The signalling equilibrium concerning credit risk has been analyzed by many authors. In the case of spot loan markets, signals analyzed include collateral (Bester 1985; and Chan and Thakor 1987), loan size (Milde and Riley 1988), and various loan contract variables (Besanko and Thakor 1987). In the case of loan commitment markets, Duan and Yoon (1993) analyze line size as a signal.

levels of investment in order to signal their true credit qualities. Since all borrowers except those with the lowest quality are bound to over-invest in the process, firms' aggregate investments and hence banks' credit supply are likely to be greater in this signalling equilibrium than in the previous one.

Social welfare comparison between the two regimes depends also on the informational scenarios. In the full information scenario, social welfare is clearly greater in the deregulated regime than in the regulated regime. In the asymmetric information scenario, however, whether or not social welfare is greater in the former is in general ambiguous and depends on the nature of firms' investment projects.

The rest of the paper is organized as follows. In Section II, the basic model of a credit market is provided. In Section III, we discuss the regulated loan rate regime. The full information scenario is discussed in Sub-section A, and the asymmetric information scenario and the associated signalling equilibrium are discussed in Sub-section B. In Section IV, we discuss the deregulated loan rate regime. The full information scenario together with the socially optimal investment level is analyzed in Sub-section A, and the asymmetric information scenario and the associated informationally consistent signalling equilibrium are discussed in Sub-section B. Section V concludes. All proofs are collected in Appendix.

II. The Basic Model

We consider one period economy with two event dates, $t=0$ and $t=1$. There are firms, banks and a regulator in the economy, all of whom are risk-neutral. Each firm has an investment project which yields, at $t=1$, per dollar return α if successful and 0 if unsuccessful. The project's success probability, θ, may differ across firms and determines their credit qualities, or equivalently their types. The distribution function governing firms in terms of θ, $F(\theta)$, is assumed to be uniform on the interval [0, 1] for simplicity.[4]

[4]We basically abstract the complex impacts of the firm type distribution on the regulator's optimal choice of the common loan rate as well on social welfare.

A firm finances its investment project entirely by a bank loan.[5] Upon a loan request, a bank either lends what the firm demands by charging an appropriate loan rate or rejects it completely. Accordingly the firm's investment amount, if any, equals the size of its loan demand. To execute the investment project, however, the firm (or its owner/manager) must expend an effort the cost of which increases with investment amount, Q, at an increasing rate. We let this cost be $\gamma Q^2/2$, where γ is a positive constant.[6] We let r (>1) be one plus risk-free interest rate at $t=0$. Banks pay out this rate to their depositors at $t=1$ regardless of borrowers' default.

We now consider two alternative regulatory regimes. In the regulated loan rate regime, the regulator fixes a loan rate which all banks must apply to their borrowers. However, banks are free to select to whom they grant loans. Given the loan rate, therefore, banks are able to earn some positive expected profits by lending only to firms with some better credit qualities, i.e. they can earn a regulatory rent. The regulator chooses a common loan rate in order to maximize social welfare.

In the deregulated loan rate regime, the regulator allows banks to freely choose loan rates. Since competition would be in full force in this regime, the loan rates banks charge are bound to reflect borrowers' credit qualities. In equilibrium, banks earn only zero expected profit.

Firms' realized payoffs at $t=1$ (α and 0) are assumed to be publicly observed. However, the observability of a firm's success probability, θ, will depend on our informational assumptions to be specified below.

When a firm of type θ borrows an amount Q at a loan rate R from a bank, its value from investment/financing is

$$V(\theta;\, Q,\, R) = \frac{\theta(\alpha - R)Q}{r} - \frac{\gamma Q^2}{2}. \tag{1}$$

If the firm succeeds in the project at $t=1$, it earns αQ and successfully pays out RQ to the bank. On the other hand, if it fails, it earns nothing and accordingly pays out nothing to the

[5] This assumption eliminates the potential confounding effects of equity financing in the signalling equilibriums that follow.

[6] This can be interpreted as the owner/manager's opportunity cost of effort which is nonremunerable.

bank.[7] The firm expends the effort in the value of $\gamma Q^2/2$ at $t=0$.

The bank's value from such a loan is

$$\Pi(P(Q);\ R,\ Q) = \frac{P(Q)RQ}{r} - Q\ , \tag{2}$$

where $P(Q)$ represents the bank's estimation of the firm's credit quality to be specified below. In (2), the bank gets paid RQ if the firm is successful, but zero otherwise. Bank depositors will get paid rQ in all states.[8]

III. Regulated Loan Rate Regime

In this section, the regulator is assumed to require banks to apply a common loan rate to all borrowers. This loan rate is set by the regulator who maximizes the social welfare to be defined below. Given the loan rate, however, banks are free to choose to whom they grant loans. That is, a bank may have loan demands of some qualified satisfied and reject those of others. We discuss full and asymmetric informational scenarios in Sub-sections *A* and *B*, respectively.

A. Full Information

In this sub-section the true value of θ is assumed to be public information. That is, $P(Q)=\theta$. We first discuss the firm's loan demand decision and then the bank's loan supply decision. We then discuss the regulator's problem of determining the common loan rate.

Given R, a firm of type θ would choose $Q \geq 0$ to maximize V in (1). Solving this problem yields its optimal investment (and loan demand) given R as

$$Q^{I}(\theta,\ R) = \frac{\theta(\alpha - R)}{\gamma r}\ . \tag{3}$$

[7] In both equations (1) and (2), the cost of capital equals the risk-free rate due to global risk neutrality.

[8] It is viewed that the bank pays out πQ to the deposit insurance at $t=0$, where $\pi = 1 - P(Q)$, and the deposit insurance pays off rQ to the depositors at $t=1$ if the loan defaults.

Assuming that this firm borrows Q^{I} at R, we substitute (3) into (1) to get the firm's optimal value as

$$V^{\mathrm{I}}(\theta\,;\,Q^{\mathrm{I}},\,R) = \frac{\theta^2(\alpha - R)^2}{2\gamma r^2}\,. \tag{4}$$

Note that $V^{\mathrm{I}} \geq 0$ for all $\theta \geq 0$ and $R \leq \alpha$, where the equality holds for $\theta = 0$, $R = \alpha$ or both.

The bank's value from lending Q to the firm is

$$\Pi^{\mathrm{I}}(\theta\,;\,Q,\,R) = \left(\frac{\theta R}{r} - 1\right) Q \gtreqless 0 \quad \text{for} \quad \theta \gtreqless \frac{r}{R} \equiv \theta^{\mathrm{I}}\,. \tag{5}$$

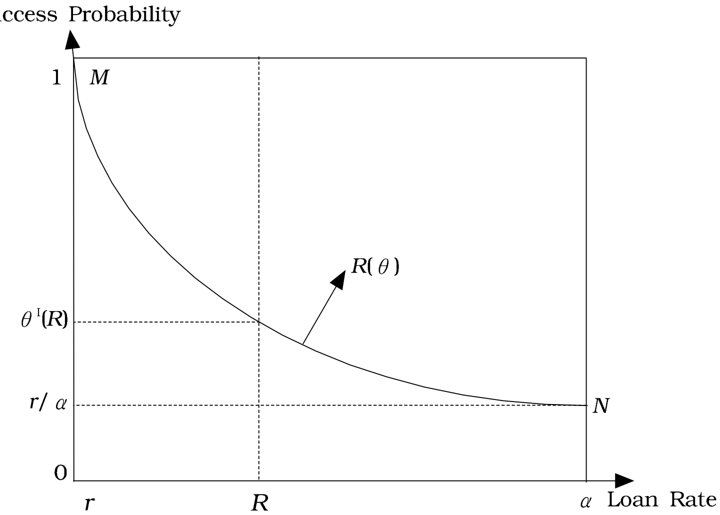
Thus, the bank will have the loan demands of qualified firms (with $\theta \geq \theta^{\mathrm{I}}$) satisfied, but will reject those of unqualified firms (with $\theta < \theta^{\mathrm{I}}$).[9]

Figure 1 helps visualizing both the firm's value and the bank's in the regulated regime with full information. The downward-sloping line *MN* represents the schedule of minimum acceptable loan quality, θ^{I}, as a function of the common loan rate. At R, a bank earns positive (zero) expected profits from lending to the qualified firms located above (on) the line *MN*. Thus, it will have their loan demands satisfied. However, it earns negative expected profits from lending to the unqualified firms located below the line. It will therefore reject their loan demands.[10]

To discuss the regulator's problem of choosing R, we first define the social welfare as the sum of the aggregate firm value and the aggregate bank value. Assigning an equal weight for both as well as across firm types, we can write the social welfare as

[9] In the literature, credit rationing in the form of a bank's rejecting a customer's loan demand completely has been referred to as 'customer rationing', while that in the form of a bank's granting only part of what a customer demands has been referred to as 'size rationing'. See Keeton (1979). The current paper deals with the former.

[10] Note that with full information it is possible for a bank and a firm to engage in some side-payments. For instance, in order to attract a firm with a higher quality, a bank may offer a pecuniary or nonpecuniary benefit, say by lowering compensating balance and collateral requirements or providing some bribery. An unqualified firm may also offer a similar benefit to a bank in order to have its loan demand fulfilled. Since allowing such a possibility would clearly contradict the intent of the common loan rate regulation, we assume that it can be ruled out by the regulator.



Note: Given $R<\alpha$, the firm's value from investment is positive (zero) for $\theta>(=)0$, while the bank's value from lending is positive (zero, negative) above (on, below) line *MN*. This line corresponds to $R(\theta)$ in (15).

FIGURE 1

THE FIRM'S VALUE AND THE BANK'S VALUE IN $\{\theta, R\}$

$$W^{\mathrm{I}}(R) = \int_{\theta^{\mathrm{I}}}^{1} \{V^{\mathrm{I}}(\theta;\, Q^{\mathrm{I}}(\theta,\, R),\, R) + \Pi^{\mathrm{I}}(\theta;\, Q^{\mathrm{I}}(\theta,\, R),\, R)\} d\theta$$

$$= \int_{\theta^{\mathrm{I}}}^{1} Q^{\mathrm{I}} \left(\frac{\theta\alpha}{r} - \frac{\gamma Q^{\mathrm{I}}}{2} - 1 \right) d\theta \,. \tag{6}$$

We now define $Q^{\mathrm{o}}(\theta)$ as the socially efficient investment level for the firm of type θ. Since this level maximizes the value of the integrand in (6) for a given θ, we get $Q^{\mathrm{o}}(\theta)=\mathrm{Max}[0,\ (\theta\alpha-r)/\gamma r]$. Comparing this level with the amount in (3), we see that all firms of type $\theta>\theta^{\mathrm{I}}$ invest less than what is socially efficient, i.e. $Q^{\mathrm{I}}(\theta) < Q^{\mathrm{o}}(\theta)$. This under-investment result arises because their loans are over-priced at *R*.[11]

[11]What if the regulator prohibits price discrimination across different borrowers by a bank but allows price competition among banks? To see the consequences of such a regulation, consider *R* as the single loan rate chosen by a bank and offered to all of its borrowers irrespective of their credit qualities. Suppose this bank chooses to lend to firms with $\theta\geq\theta^{\mathrm{I}}$. Then

***Lemma* 1**

In the regulated loan rate regime with full information the common loan rate that maximizes the social welfare in (6), R^{I}, exists and satisfies $r<R^{I}<\alpha$.

***Proof*:** See Appendix.

The welfare-maximizing loan rate is determined by the following trade-off. On one hand, as R increases, firms which were previously unqualified become qualified, and thus can have their loan demands fulfilled. This adds to social welfare. On the other hand, an increase in R leads those already qualified to reduce their investments. This reduces social welfare as their investments have already been lower than the socially efficient level.

Lemma 1 indicates that, with $R^{I}>r$, some firms cannot have their loan demands fulfilled. Among them, some could have contributed positively to social welfare had they been funded even at a higher loan rate. They are of type $\theta\in[\theta^{o}, \theta^{I}(R^{I})]$, where $\theta^{o}\equiv r/\alpha$.[12] Here we see that the common loan rate is indeed the source of welfare loss.

The optimal value of the social welfare in the regulated loan rate regime with full information, W^{I}, can be obtained by substituting R^{I} into (6).

B. Asymmetric Information

In this sub-section, a firm's success probability, θ, is assumed to be its private information. Upon a loan request from a firm, a bank charges R set by the regulator, and applies a loan granting probability, $Z(P(Q))$. The bank computes this probability by using the estimation function, $P(Q)$, which maps the size of loan demand to firm type.

another bank can profitably lend only to firms with $\theta\geq\theta'(>\theta^{I})$ by offering another loan rate lower than R. Competition among banks then keeps on lowering the loan rate down with more and more firms being rejected until when loans are made only to firms with $\theta=1$ at $R=r$. A market failure as in the Akerlof's (1970) market for lemons. Thus, if there exists a non-trivial equilibrium under such a regulation, the regulator must also fix a loan rate for the market. The regulator's interference with the market by prohibiting intrabank price discrimination creates a need for another of self-fixing the loan rate.

[12]Note that $V^{I}(\theta)+\Pi^{I}(\theta)\geq 0$ for $\theta\geq\theta^{o}$.

Given R and $Z(P(Q))$, the value of the firm of type θ is

$$V^{II}(\theta;\ R,\ Q)=Z(P(Q))\left(\frac{\theta(\alpha-R)Q}{r}-\frac{\gamma Q^2}{2}\right). \tag{7}$$

The bank's value from lending Q to this firm is

$$\Pi^{II}(P(Q);\ R,\ Q)=Z(P(Q))\left(\frac{P(Q)RQ}{r}-Q\right). \tag{8}$$

To see why the estimation function has Q as an argument, consider two firms of high and low type. In order to distinguish itself from the low type and hence to increase its loan granting probability, the high type firm would increase its investment (loan demand) up to the level which renders the low type's mimicking strategy marginally unprofitable. This is possible because the marginal profit associated with an increase in investment is greater for the high type than for the low type.

To formalize this concept, we define the signalling equilibrium given R, as a triplet of functions $\{Q^{II}(\theta),\ Z(P),\ P(Q)\}$ such that, for all θ,

$$Q^{II}(\theta)\in\operatorname{argmax}\ V^{II}(\theta;\ R,\ P(Q)), \tag{9}$$

$$Z(P)=\begin{cases}1\\0\end{cases}\quad \text{for } P(Q)\begin{cases}\geq\\<\end{cases}\theta^{I},\quad \text{and} \tag{10}$$

$$P(Q)\geq(<)\,\theta^{I}\qquad \text{for } Q\geq(<)Q^{*}, \tag{11}$$

where Q^* is a hurdle amount. Condition (9) is the incentive compatibility condition for the firm of type θ. That is, given $Z(P)$ and $P(Q)$, it is optimal for the firm to choose Q^{II}. Condition (10) describes the bank's loan granting policy. The bank observes Q chosen by the firm, computes $P(Q)$, and assigns $Z(P)$. If $Q\geq Q^*$, the firm is viewed as qualified. Thus, the bank assigns $Z=1$ and has its loan demand fulfilled. If $Q^{II}<Q^*$, on the other hand, the firm is viewed as unqualified. Thus, the bank assigns $Z=0$ and rejects its loan application. Condition (11) states the informational consistency required to separate between the qualified and unqualified firms. Note here that since the loan rate is fixed, the bank only needs to know whether a firm is of type $\theta\geq\theta^{I}$ or not. Also note that once this information is revealed, firms no longer have an incentive to misrepresent their exact types.

***Proposition* 1**
In the regulated loan rate regime with asymmetric information, a signalling equilibrium exists for $R \in (r, \alpha)$ if and only if :

$$Q^{\mathrm{II}}(\theta) = \begin{cases} \mathrm{Max}[Q^*, Q^{\mathrm{I}}(\theta)] \\ 0 \end{cases} \text{for } \theta \begin{Bmatrix} \geq \\ < \end{Bmatrix} \theta^{\mathrm{I}}, \tag{12}$$

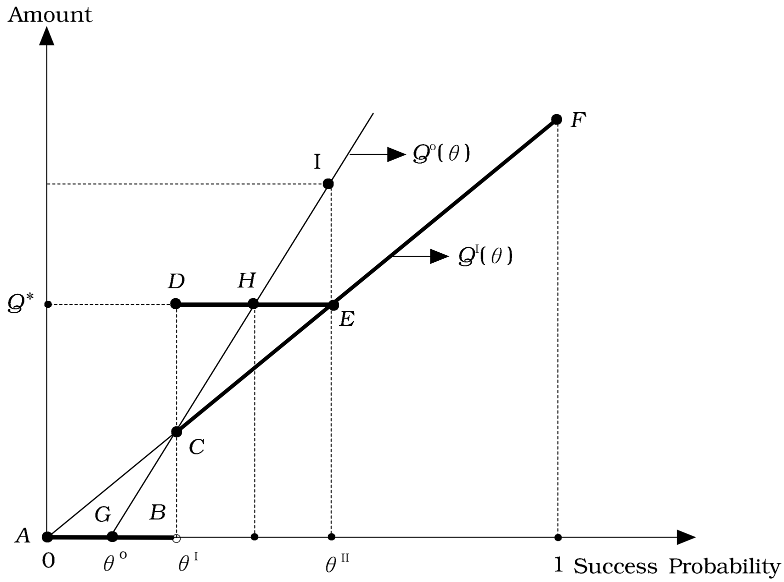
where $Q^* = \mathrm{Min}[(\alpha - R)/\gamma r, \; 2(\alpha - R)/\gamma R]$.

Proof: See Appendix.

The hurdle amount Q^* leads the firm of type θ^{I} to just break even, i.e. it represents the upper limit for investment for the threshold firm at R. To see the intuition behind the demand behavior in (12), note that the upper limit increases with the success probability. This holds true because the marginal productivity of effort associated with an increase in investment increases with the success probability. Therefore qualified firms would increase their loan demands at least up to the hurdle amount in order to distinguish themselves from the unqualified. They anticipate that the unqualified will not mimic their investment behavior and that their loan granting probability will become one. This anticipation is confirmed in equilibrium as banks read the signals by firms correctly. Competition in the loan market then forces banks to adopt the loan granting policy in (10). Accordingly the loan demands of the qualified are fulfilled but those of the unqualified are not.

This signalling equilibrium is indeed a Nash equilibrium in that given R, there exists no alternative offer which, if made by a bank, would result in an increase in its value from that of the equilibrium offer. As competition among banks takes place only in terms of the loan granting probability, any offer different from (10) reduces the bank's profit.

To compare the aggregate investment levels in the two informational scenarios, consider the same R for both. From (12) one can then see that the aggregate investment level is clearly higher in the asymmetric information scenario than in the full information scenario. However, this result does not in general hold as the welfare-maximizing common loan rate can be higher in the former.



Note: Given R, line segments AB and CEF represent firms' investments in the full information equilibrium. Line segments AB and DEF represent those in the signalling equilibrium under the assumption that $\theta^{II} < 1$ or $Q^* = 2(\alpha - R)/\gamma R$. Line $GCHI$ indicates the socially efficient investment level.

Figure 2

Firms' Investments in the Regulated Loan Rate Regime

Figure 2 depicts firms' investment behavior in the two informational scenarios in the regulated loan rate regime. For an illustrative purpose, we assume the same loan rate for both scenarios and that $\theta^{II} < 1$. Line $ACEF$ represents firms' loan demands in the full information scenario, whereas line segments AB and CEF represent their investments or banks' credit supply. Line segments AB and DEF represent those in the signalling equilibrium in which every firm's loan demand is fulfilled. If a firm is of type $\theta \in [\theta^I, \theta^{II})$, then its investment is greater in the signalling equilibrium than in the full information equilibrium. On the other hand, if a firm is of type $\theta \in [\theta^{II}, 1]$, then its investment remains the same. Thus, the aggregate investment level is higher in the asymmetric information scenario than in the full information scenario. However, this result could change if the common loan rate turns out to be higher in the asymmetric information scenario. Further, the higher aggregate investment level in the signalling equilibrium does not necessarily imply a greater social welfare. This is because the welfare loss arising from the over-investments by firms of type θ that is equal to and slightly greater than θ^{I} could exceed the welfare loss associated with the under-investments in the full information equilibrium. In the figure, Q^* deviates further from $Q^{\mathrm{o}}(\theta)$ than $Q^{\mathrm{I}}(\theta)$ at and near θ^{I}.

Turning to the regulator's problem of setting R for the signalling equilibrium, we write the social welfare in the asymmetric information scenario given R as

$$W^{\mathrm{II}}(R) = \int_{\theta^{\mathrm{I}}}^{1} \{V^{\mathrm{II}}(\theta;\, Q^{\mathrm{II}},\, R) + \Pi^{\mathrm{II}}(\theta;\, Q^{\mathrm{II}},\, R)\} d\theta\,. \tag{13}$$

Using (10), (11) and (12), we rewrite (13) as

$$W^{\mathrm{II}}(R) = \int_{\theta^{\mathrm{I}}}^{\theta^{\mathrm{II}}} Q^* \left(\frac{\theta\alpha}{r} - \frac{\gamma Q^*}{2} - 1 \right) d\theta + \int_{\theta^{\mathrm{II}}}^{1} Q^{\mathrm{I}}(\theta) \left(\frac{\theta\alpha}{r} - \frac{\gamma Q^{\mathrm{I}}(\theta)}{2} - 1 \right) d\theta\,. \tag{14}$$

In (14) note that if $R<2r$, then θ^{II} equals 1, Q^* equals $(\alpha - R)/\gamma r$ and the second integral vanishes.

***Lemma* 2**

In the regulated loan rate regime with asymmetric information, the common loan rate that maximizes the social welfare in (14), R^{II}, exists and satisfies $r<R^{\mathrm{II}}<\alpha$.

Proof: See Appendix.

To understand the regulator's problem of choosing R^{II}, we consider the effects of a change in the common loan rate on three different groups of firms. The first group consists of firms of type $\theta \in [\theta^{\mathrm{II}}, 1]$. Since these firms invest $Q^{\mathrm{I}}(\theta)$, which is smaller than $Q^{\mathrm{o}}(\theta)$, in the signalling equilibrium, an increase in the loan rate would reduce their investments. This yields a decrease in the social welfare. The second group consists of firms of type $\theta \in [\theta^{\mathrm{I}}, \theta^{\mathrm{II}})$. While these firms invest Q^* in the signalling equilibrium, an increase in the loan rate would reduce Q^* itself and hence θ^{II}. However, since Q^* is either greater or less than $Q^{\mathrm{o}}(\theta)$, the net effect on the social welfare is in general ambiguous. The last group consists of firms of type $\theta \in [0, \theta^{\mathrm{I}})$. An increase in the loan rate would lower θ^{I}, leading some marginal firms to newly invest Q^* and hence adding to the social welfare. With the remaining firms, however, it would not have any impact as their loan demands continue to be rejected.

The optimal value of the social welfare, W^{II}, can be determined by substituting R^{II} into (14). Note that while the signalling mechanism partly corrects under-investments by some firms, it instead results in over-investments by other firms. Thus, the net effect on the social welfare is ambiguous.

IV. Deregulated Loan Rate Regime

In this section, we analyze the deregulated loan rate regime where the regulator allows individual banks to freely charge different loan rates on the basis of borrowers' credit qualities. Since competition among banks would be in full force in this regime, banks are bound to charge the risk-adjusted loan rates that lead them to break even with each and every loan applicant. We discuss the full information and asymmetric information scenarios in Sub-sections *A* and *B*, respectively.

A. Full Information

Full information is re-assumed here. Accordingly, the risk-adjusted loan rate schedule becomes

$$R(\theta) = \frac{r}{\theta} \qquad \text{for all } \theta \in [0, 1]. \tag{15}$$

Given that this loan rate schedule is downward sloping in θ and that α is a constant, we have

$$R(\theta) \gtreqless \alpha \qquad \text{for } \theta \lesseqgtr \frac{r}{\alpha} \equiv \theta^{\circ}, \tag{16}$$

where θ° is the minimum acceptable loan quality defined earlier. Note that since a bank can only charge Min$[R(\theta), \alpha]$ to a firm, it breaks even only if it lends to firms of type $\theta \geq \theta^{\circ}$ but incurs a loss if it lends to those of type $\theta < \theta^{\circ}$.

Consider a firm of type $\theta \geq \theta^{\circ}$. Using (3) and (15), we get the optimal investment for this firm as

$$Q^{\mathrm{III}}(\theta)=\frac{\theta\alpha-r}{\gamma r}. \tag{17}$$

This equals the efficient investment level defined earlier, i.e. $Q^{\mathrm{III}}(\theta)=Q^{\mathrm{o}}(\theta)$. Note that when a bank charges the risk-adjusted loan rate given by (15), it only earns zero expected profit and hence ensures efficient investment on the part of the firm. We get $Q^{\mathrm{III}}(\theta)=0$ for firms of type $\theta<\theta^{\mathrm{o}}$.[13]

Substituting (15) and (17) into (1), we obtain the firm's optimal value in this scenario as

$$V^{\mathrm{III}}(\theta)=\mathrm{Max}\left[\frac{(\theta\alpha-r)^2}{2\gamma r^2},\ 0\right]. \tag{18}$$

Given that banks earn zero expected profit and each firm earns $V^{\mathrm{III}}(\theta)$ in (18), the social welfare in this scenario can be written as

$$W^{\mathrm{III}}(\theta)=\int_{\theta^{\mathrm{o}}}^{1}V^{\mathrm{III}}(\theta)d\theta. \tag{19}$$

***Proposition* 2**

In the full information scenario, aggregate investments as well as social welfare increase with the loan rate deregulation.

Proof: See Appendix.

Proposition 2 establishes that in the full information scenario the regulator could increase social welfare by giving up the loan rate regulation. Such a deregulation would lead banks to apply competitively determined risk-adjusted loan rates to individual borrowers. Consequently, the bank's value would decrease (to zero), whereas the firm's value would increase. Since the magnitude of the increase in the latter is greater than that of the decrease in the former, social welfare increases.

This increase in social welfare is due to the fact that each firm increases its investment to the efficient level. Firms that were qualified at R^{I} would increase their investments in the deregulated loan rate regime because $R(\theta)\leq R^{\mathrm{I}}$ for them. As a result, their values as well as social welfare would increase. On the other hand,

[13]These firms would not demand any loan simply because the risk-adjusted loan rates are too expensive for them. This phenomenon is sometimes referred to as 'price rationing' in the literature.

firms that were not qualified at R^{I} (i.e. firms of type $\theta \in [\theta^{o}, \theta^{\mathrm{I}}]$) would become newly qualified at $R(\theta)$. Thus, social welfare further increases as these firms can have their loan demands financed.

B. Asymmetric Information

In this sub-section, we return to the assumption that θ of a firm is its private information. We are interested in establishing the signalling equilibrium which works differently from the one discussed in Section III. Importantly, the signalling equilibrium in this regime is subject to competitive rationality of the credit market. That is, the contractual loan rate for a firm of type θ is determined as

$$R(r, P(Q)) = \frac{r}{P(Q)}, \tag{20}$$

where $P(Q)$ is the market's estimation of θ which is based on the firm's loan demand Q. A bank expects to earn zero profit by charging $R(r, P(Q))$ to the firm when its expectation is according to the estimation function.

The reason why the estimation function has investment amount as an argument is basically the same as that in the regulated regime. When the market suffers from asymmetric information, a firm with higher credit quality has an incentive to distinguish itself from those with lower credit qualities, anticipating that it will be charged a lower loan rate. This firm can in fact profitably increase its investment up to the level which those with lower credit qualities would find unprofitable. This is again possible because the marginal profit associated with an increase in investment increases with the firm's credit quality. However, unlike in the regulated regime, the current signalling equilibrium involves full separation of firms by way of the loan rates charged.

Following Riley (1979), an informationally consistent signalling equilibrium given $R(r, P(Q))$ is defined as a pair of functions $\{Q(\theta), P(Q(\theta))\}$ such that, for all $\theta \geq \theta^{o}$,

$$Q^{\mathrm{IV}}(\theta) \in \operatorname{argmax} V^{\mathrm{IV}}(\theta; Q, P(Q)) = \frac{\theta\{\alpha - r/P(Q)\}Q}{r} - \frac{\gamma Q^2}{2}, \text{ and} \tag{21}$$

$$P(Q^{\mathrm{IV}}(\theta)) = \theta. \tag{22}$$

Condition (21) is the incentive-compatibility condition for the firm of type θ. This firm takes $P(Q)$ as given, and chooses Q^{IV} to maximize its value. Each bank also takes $P(Q)$ as given, and applies the risk-adjusted loan rate given by (20). Condition (22) requires that the firm's optimal investment level fully reveals its success probability. This condition gives rise to $R(\theta)=r/\theta$ in equilibrium.

***Proposition* 3**

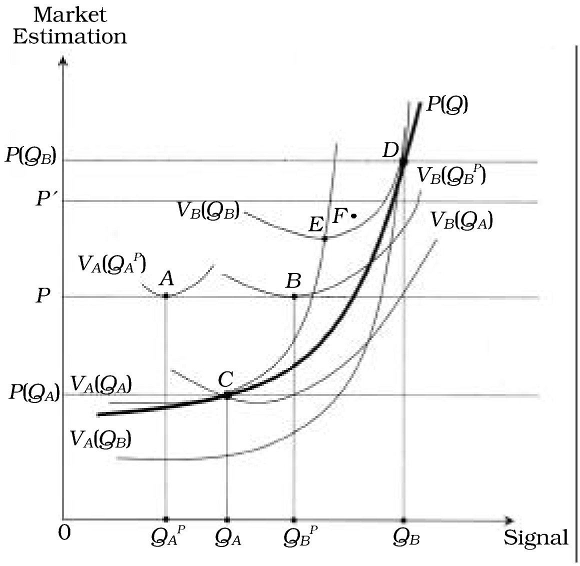
In the deregulated loan rate regime, an informationally consistent signalling equilibrium exists and satisfies: (i) $Q^{IV}(\theta)$ is strictly increasing in θ for $\theta \geq \theta^{o}$, and equals 0 for $\theta \leq \theta^{o}$; and (ii) $R(r, P(Q))$ is strictly decreasing in Q.

Proof: See Appendix.

In this signalling equilibrium, firms with successively higher θ would apply for a successively greater bank loan in order to provide a credible signal to banks. As a result, the signalling equilibrium is fully revealing among firms of type $\theta \geq \theta^{o}$. Competition among banks would then determine the risk-adjusted loan rates according to the schedule in (20). While all banks break even with each and every loan by charging such a loan rate, firms of type $\theta < \theta^{o}$ would not apply for a loan.

Figure 3 highlights the intuition behind such a signalling equilibrium, where two firms of different types, i.e. θ_A and θ_B with $\theta^{o} < \theta_A < \theta_B$, are considered. Given that the market can not observe each firm's type, we first let $P(Q)=P$, where P is a constant representing banks' estimation of the average credit quality of the two firms. By taking P as given, the two firms respectively choose Q_A^{P} and Q_B^{P} (or contracts A and B) and realize the values, $V_A(Q_A^{P})$ and $V_B(Q_B^{P})$, respectively. However, contracts A and B cannot constitute an equilibrium because of the following signalling possibility. Consider θ_B-type firm choosing Q_B. This firm might do this, anticipating that if banks read its signal correctly, then its loan would be priced according to $P(Q_B)=\theta_B$ and thus its value could increase to $V_B(Q_B)$. Meanwhile, banks will indeed go through the following inference: had this firm been truly of type θ_A, then it would have never demanded Q_B because $V_A(Q_B)<V_A(Q_A^{P})$.[14] Note that

[14]As a matter of fact, it is assumed that banks, having gone through such inferences, move first to offer the loan rate schedule in (20). And then firms move to choose the signal.



Note: The direction of an increase in the firm's value, V, is north. Bank's value is zero (negative, positive) on (above, below) the market estimations, P, P' and $P(Q)$. Subscripts A and B indicate firm types θ_A and θ_B, respectively, where $\theta^{o} < \theta_A < \theta_B$. P represents the average credit quality of the two firms of types θ_A and θ_B only. P' represents that of a continuum of firms of type $\theta \in [\theta_A, \theta_B]$.

FIGURE 3

THE COMPETITIVE SIGNALLING EQUILIBRIUM WITH INVESTMENT AS A SIGNAL

at any intersection between the two iso-value curves for type θ_A and type θ_B (e.g. point E), the former cuts through the latter from below.[15] This property holds because, given the same amount of Q, θ_B-type firm benefits more from an increase in P (a decrease in R) due to its higher success probability. Therefore it can afford to borrow more at the given P (or R). Once it reveals its type, banks could infer that the other firm is of type θ_A. As this firm chooses Q_A, it follows that $P(Q_A) = \theta_A$. Separation takes place in this credit market in that θ_B-type firm chooses contract D to earn $V_B(Q_B)$, and θ_A-type firm chooses C to earn $V_A(Q_A)$. Contracts C and D together constitute a signalling equilibrium.

[15]That is, the marginal rate of substitution between P and Q is decreasing in θ, or Riley's condition 5 holds. See the proof of Proposition 3 in Appendix.

Given the continuum of types in the current model, the estimation schedule $P(Q)$ becomes continuous like the dark line connecting contracts C and D in Figure 3 in equilibrium. As individual firms optimally choose Q given such a schedule, this schedule embraces the iso-value curves of all firms of type $\theta \geq \theta^{o}$ from outside with each tangency point occurring at $P(Q(\theta)) = \theta$. Thus, banks applying the loan rate schedule in (20) break even with each and every borrower.

For the purpose of clarification, a brief discussion of equilibrium concepts is in order. As is well known in the literature, the signalling equilibrium discussed in this sub-section is not a Nash equilibrium as the less informed agents (banks) move first by offering the loan rate schedule, $P(Q)$. Instead, it is a reactive equilibrium suggested by Riley (1979).[16] To understand this equilibrium intuitively, we again refer to Figure 3 and explicitly consider continuous types. Given the equilibrium offers associated with $P(Q)$, consider a bank offering an alternative pooling offer, say point E. Note that both θ_A-type firms and θ_B-type firms are indifferent between this offer and the original offers C and D, respectively. Also note that firms of type $\theta \in (\theta_A, \theta_B)$ strictly prefer E to those on the dark line, and therefore choose E. In this situation, if the average credit quality of all the firms choosing E is relatively high, or more specifically the average market estimation line stays higher than E (e.g. line P'), then the pooling offer is profitable. Thus, contracts C and D are no longer a Nash equilibrium.[17]

Despite the potential gains from introducing the new offer E, however, we now argue that such a defection by banks will be deterred for the following reason. With one bank offering E, another bank can take advantage of the new offer by reacting with another offer, say F. Note that this reacting offer 'skims the cream' off E in

[16]For applications of this equilibrium concept to financial markets, see John and Williams (1985), Milde and Riley (1988), and Duan and Yoon (1993) for examples.

[17]However, if the line P' stays lower than E, then the pooling contract turns out to be unprofitable, and therefore contracts C and D may constitute a Nash equilibrium. This indicates that the viability of a Nash equilibrium critically depends on the nature of the distribution function, $F(\theta)$. If $F(\theta)$ is sufficiently convex over the interval $[\theta_A, \theta_B]$, there could exist alternative offers, such as E, which yield expected profits to the offering bank. If $F(\theta)$ is sufficiently concave, however, all such offers yield expected losses. For a detailed discussion on this issue, see Riley (1979).

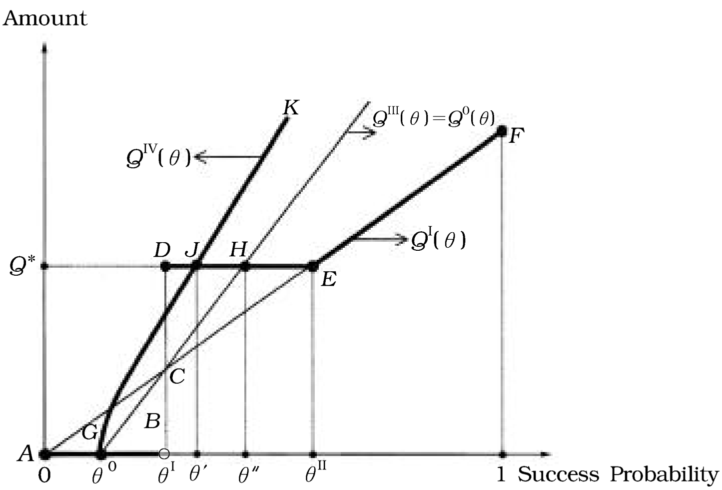
that it attracts all the higher quality firms with $\theta \geq \theta'$ and leaves only the lower quality firms with $\theta < \theta'$ to the original defector for some $\theta' \in (\theta_A, \theta_B)$. As a result, the original defector suffers a loss. Furthermore, since the reacting offer is profitable to the reactor (F is below P'), there can be no further reactions by other banks which result in a loss for the reactor. Offers superior to F may be made to bid some high quality borrowers away, however, the reactor's expected profit will never be negative. As long as there is no risk of loss on the part of the reactor, the defector's incentive to offer E disappears. The original contracts C and D then become a reactive equilibrium.

This signalling equilibrium is also dissipative. In the process of overcoming the informational problem, all firms of type $\theta > \theta^{o}$ would increase their investments beyond the full information levels (i.e. $Q^{IV}(\theta) > Q^{III}(\theta)$).[18] From the definition of $Q^{III}(\theta)$, it then follows that the social welfare in the asymmetric information scenario, W^{IV}, would be less than that in the full information scenario in this deregulated regime. While all banks break even in equilibrium, all borrowers over-invest only to realize lower values. Furthermore, the degree of over-investment increases with the credit quality, and so does the decrease in the firm's value.

We now discuss the effects of the loan rate deregulation on aggregate investments and social welfare in the asymmetric information scenario. To examine the effect on aggregate investments, observe first that whether a firm's investment level increases with the deregulation critically depends on its credit quality. To see this, we refer to Figure 4 in which line GJK illustrates $Q^{IV}(\theta)$ in (21) and line segments AB and DEF continue to illustrate $Q^{II}(\theta)$ as in Figure 2. Observe that firms of type $\theta \in (\theta^{o}, \theta^{I})$ as well as those of type $\theta \in (\theta', 1)$ would increase investments with the deregulation. However, firms of type $\theta \in (\theta^{I}, \theta')$ would do the opposite. Therefore, it is likely that aggregate investments increase, but there is a possibility that they decrease if Q^* becomes greater and point J is close to point H.[19]

[18]To see this result, consider (A9) in Appendix. Since we have $V_P > 0$ (from (R3)) and $dP/dQ > 0$ (from the last part of the proof of Proposition 3), the expression in (A9), if evaluated at $Q^{III}(\theta)$, would be positive. The concavity of V in Q (from (R6)) then gives rise to the desired result.

[19]In Figure 4, schedule $Q^{IV}(\theta)$ is depicted as a concave function to illustrate a normal case. The cross-over point J must lie to the right of



Note: Line segments *AG* and *GJK* represent firms' investments in the signalling equilibrium in the deregulated loan rate regime. Line segments *AB* and *DEF* represent those in the regulated loan rate regime.

FIGURE 4

COMPARISON OF FIRMS' INVESTMENTS IN THE TWO SIGNALLING EQUILIBRIUMS

We next compare firms' investments in each of the two signalling equilibriums with the socially efficient level. This will help us compare the social welfare levels in the two regimes. Observe that all borrowers tend to over-invest in the deregulated regime compared with the efficient level. However, some firms (of type $\theta \in [\theta^{I}, \theta'']$) over-invest but others under-invest in the same sense in the regulated regime. Thus, whether the social welfare in the deregulated signalling equilibrium would be greater than that in the regulated signalling equilibrium is in general ambiguous. While both signalling equilibriums entail efficiency losses, the nature and extent of the losses are different. The outcome of this comparison depends on the magnitude of the signalling costs associated with the over-investments in the former relative to that associated with the under-investments in the latter where the signalling possibility has an offsetting effect. Thus, the outcome again depends on the

point *D*, where $V^{II}(\theta^{I}; R^{II}, Q)=0$, so that $V^{IV}(\theta)>0$ holds for all $\theta > \theta^{o}$.

nature of firms' investment project and also potentially on the distribution governing them in terms of the credit quality.

V. Conclusion

This paper analyzes credit market equilibriums in both the regulated and deregulated loan rate regimes. In the regulated loan rate regime, the regulator sets a common loan rate to maximize social welfare and requires banks to charge it to all borrowers regardless of their credit risks. When banks have full information regarding the credit risks, on one hand, they will have loan demands of qualified firms satisfied but reject those of unqualified ones. In equilibrium, banks earn some regulatory rents and firms invest less than what is socially efficient.

When banks do not have full information, on the other hand, a signalling equilibrium is shown to exist given the common loan rate set by the regulator. In equilibrium, better credit quality firms increase their loan demands at least up to a hurdle amount in order to signal their true types to banks. In the process, some medium credit quality firms are bound to increase their investments over and above the socially efficient level. Firms with high credit qualities invest the same amount as in the full information equilibrium. Banks continue to earn some regulatory rents in this scenario as competition in terms of loan rates is not allowed. Due to the over-investments by some of the medium credit quality firms, this signalling equilibrium is dissipative.

In the deregulated loan rate regime, the regulator allows banks to freely choose their own loan rates charged to individual borrowers. As competition among banks would be in full force in this regime, the loan rates banks charge would be fully risk-adjusted and lead them to break even. In the full information scenario, on one hand, banks will have each and every loan demand satisfied and hence credits will no longer be rationed. As a result, social welfare would be maximized as the risk-adjusted loan rate leads each borrower to invest the socially efficient level.

In the asymmetric information scenario, on the other hand, an informationally consistent signalling equilibrium is shown to exist. This signalling equilibrium is also dissipative since firms tend to over-invest. Firms with successively higher credit qualities would successively increase their investments in order to credibly signal their true qualities to banks.

In the full information scenario, social welfare is shown to be unambiguously greater in the deregulated regime than in the regulated regime. This is because all borrowers tend to under-invest in the regulated regime, but invest the socially efficient level in the deregulated regime. However, in the asymmetric information scenario, the effects on social welfare in the two regimes cannot be unambiguously compared. While the loan rate deregulation partly corrects the under-investment problem associated with the regulated loan rate, the need for overcoming the informational problem could lead firms to over-invest in the deregulated regime.

Appendix

***Proof of Lemma* 1**: Substituting (3) into (6) yields

$$W^{I}(R) = \int_{\theta^{I}}^{1} \frac{\theta(\alpha - R)}{2\gamma r^{2}} \{\theta(\alpha + R) - 2r\} d\theta.$$

Using the Leibnitz's rule, we differentiate this to get the following first-order condition:

$$\frac{dW^{I}(R)}{dR} = -\frac{1}{\gamma r^{2}} \int_{\theta^{I}}^{1} \theta(\theta R - r) d\theta + \frac{(\alpha - R)^{2} r}{2\gamma R^{4}} = 0. \qquad \text{(A1)}$$

Here the second derivative of $W^{I}(R)$ can be shown to be negative given that $R \leq \alpha$. Further, the derivative in (A1), evaluated at $R=r$ and $R=\alpha$, becomes positive and negative, respectively. Thus, R^{I} which is the unique solution of (A1) satisfies the desired property.

Q.E.D.

***Proof of Proposition* 1**: (a) To prove "necessity", we show that conditions (9), (10) and (11) imply condition (12). We divide between two cases.

Case (i): Firms of type $\theta < \theta^{I}$. First, suppose that such a firm demands $Q<Q^{*}$. Then, we have $P< \theta^{I}$ from (11) and hence $Z=0$ from (10). Thus, $V^{II}=0$ from (7). This implies that $Q^{II}(\theta)=0$. Next, suppose that it demands $Q \geq Q^{*}$. We then have $P \geq \theta^{I}$ and $Z=1$ from conditions (11) and (10), respectively. Thus, from (7) we get

$$V^{\mathrm{II}}=\frac{\theta(\alpha-R)Q}{r}-\frac{\gamma Q^2}{2}. \tag{A2}$$

Note that this value non-increases in Q for $Q\geq Q^*$, $R\in[r,\alpha]$ and $\theta<\theta^{\mathrm{I}}$. Thus it has a maximum value at Q^*. Since the loan amount must be nonnegative, it follows that $Q^{\mathrm{II}}(\theta)=0$.

Case (ii): Firms of type $\theta\geq\theta^{\mathrm{I}}$. First, suppose that such a firm demands $Q<Q^*$. From conditions (10) and (11), we get $V^{\mathrm{II}}=0$. Next, suppose that it demands $Q\geq Q^*$. We need to consider two cases. In the case of firms of type $\theta\in[\theta^{\mathrm{I}},\ \theta^{\mathrm{II}}]$, where $\theta^{\mathrm{II}}=2r/R<1$, we get $V^{\mathrm{II}}(\theta;\ Q',\ R)<V^{\mathrm{II}}(\theta;\ Q^*,\ R)$ for $Q'>Q^*$. We also get $V^{\mathrm{II}}(\theta;\ Q^*,\ R)\geq 0$, where the equality holds only for $\theta=\theta^{\mathrm{I}}$. Thus, such a firm would demand Q^*. In the case of firms of type $\theta\in[\theta^{\mathrm{II}},\ 1]$, we get $V^{\mathrm{II}}(\theta;\ Q^{\mathrm{I}},\ R)\geq V^{\mathrm{II}}(\theta;\ Q^*,\ R)$, where the equality holds only for $Q^{\mathrm{I}}=Q^*$. Thus, such a firm would demand Q^{I} given by (3) with $Q^{\mathrm{I}}\geq Q^*$ for $\theta\geq\theta^{\mathrm{II}}$, where the equality holds only for $\theta=\theta^{\mathrm{II}}$. Note that if $2r/R>1$, then θ^{II} becomes 1 and the last case disappears.

(b) To prove "sufficiency", we show that condition (12) implies conditions (9), (10) and (11). First note that condition (12) yields that all the unqualified firms would not demand a loan, but all the qualified firms would demand $Q\geq Q^*$. If the bank reads these signals correctly, then it uses the estimation function in (11) and optimally employs the loan granting policy in (10). This is because it would otherwise either lose profitable loan opportunities (in case when $Z(P)<1$ for $P\geq\theta^{\mathrm{I}}$) or incur losses (in case when $Z(P)>0$ for $P<\theta^{\mathrm{I}}$). Condition (9) is immediate from conditions (10), (11) and (12).

Q.E.D.

Proof of Lemma 2: Using the definitions of Q^* in (12) and $Q^{\mathrm{I}}(\theta)$ in (3), we can rewrite (14) as

$$W^{\mathrm{II}}(R)=\int_{\theta^{\mathrm{I}}}^{\theta^{\mathrm{II}}}\frac{2\alpha(\alpha-R)}{\gamma rR^2}(\theta R-r)d\theta+\int_{\theta^{\mathrm{II}}}^{1}\frac{(\alpha-R)}{2\gamma r^2}\theta\{\theta(\alpha+R)-2r\}d\theta.$$

Note that the second integral vanishes when it holds that $R\leq 2r$. Using the Leibnitz's rule, we differentiate $W^{\mathrm{II}}(R)$ to get

$$\frac{dW^{\mathrm{II}}(R)}{dR}=-\frac{2\alpha}{\gamma rR^3}\int_{\theta^{\mathrm{I}}}^{\theta^{\mathrm{II}}}(\theta\alpha R-2r\alpha+rR)d\theta$$
$$-\frac{1}{\gamma r^2}\int_{\theta^{\mathrm{II}}}^{1}\theta(\theta R-r)d\theta. \tag{A3}$$

In addition, we get

$$\frac{d^2W^{\mathrm{II}}(R)}{dR^2} = \frac{4\alpha}{\gamma rR^4}\int_{\theta^{\mathrm{I}}}^{\theta^{\mathrm{II}}}(\theta\alpha R-3r\alpha+rR)d\theta$$

$$-\frac{2\alpha}{\gamma rR^3}(\theta^{\mathrm{II}}\alpha R-2r\alpha+rR)\frac{d\theta^{\mathrm{II}}}{dR}+\frac{2r\alpha}{\gamma R^5}(\alpha-R) \qquad \text{(A4)}$$

$$-\frac{1}{\gamma r^2}\int_{\theta^{\mathrm{II}}}^{1}\theta^2 d\theta+\frac{\theta^{\mathrm{II}}}{\gamma r^2}(\theta^{\mathrm{II}}R-r)\frac{d\theta^{\mathrm{II}}}{dR}.$$

We now evaluate the two derivatives in (A3) and (A4), respectively, at $R=r$ and $R=\alpha$. At $R=r$, note that $\theta^{\mathrm{I}}=\theta^{\mathrm{II}}=1$ and that as long as R stays close to r, θ^{II} remains fixed and thus $d\theta^{\mathrm{II}}/dR=0$. Thus, we see that the derivative in (A3) becomes zero and that in (A4) becomes positive. This implies that $R=r$ is a local minimum. At $R=\alpha$, on the other hand, we divide between two cases. In the case of $\alpha\leq 2r$, we have $\theta^{\mathrm{II}}=1$ since $R\leq\alpha$, and therefore $d\theta^{\mathrm{II}}/dR=0$. In the case of $\alpha>2r$, we have $\theta^{\mathrm{II}}=2r/R=2r/\alpha$ and therefore $d\theta^{\mathrm{II}}/dR=-2r/R^2=-2r/\alpha^2$. Using these results, we can see that in both cases the two derivatives in (A3) and (A4) become both negative. Since $W^{\mathrm{II}}(\theta)$ is continuous in R, there must exist at least one R^{II} that gives rise to zero value for the derivative in (A3) and satisfies the desired property.

Q.E.D.

***Proof of Proposition* 2**: Let E^{I} and E^{III} be the aggregate investment levels in the regulated and deregulated regimes, respectively. Then,

$$E^{\mathrm{III}}-E^{\mathrm{I}}=\int_{\theta^0}^{1}Q^{\mathrm{III}}(\theta)d\theta-\int_{\theta^{\mathrm{I}}(R^{\mathrm{I}})}^{1}Q^{\mathrm{I}}(\theta,R^{\mathrm{I}})d\theta$$

$$=\int_{\theta^{\mathrm{I}}(R^{\mathrm{I}})}^{1}\{Q^{\mathrm{III}}(\theta)-Q^{\mathrm{I}}(\theta,R^{\mathrm{I}})\}d\theta+\int_{\theta^0}^{\theta^{\mathrm{I}}(R^{\mathrm{I}})}Q^{\mathrm{III}}(\theta)d\theta>0. \qquad \text{(A5)}$$

We obtain the sign as follows: For the first integral in the above equation, we get using (3) and (17)

$$Q^{\mathrm{III}}(\theta)-Q^{\mathrm{I}}(\theta,R^{\mathrm{I}})=\frac{1}{\gamma r}(\theta R^{\mathrm{I}}-r)>0 \text{ for all } \theta\geq\theta^{\mathrm{I}}(R^{\mathrm{I}}).$$

The second integral is also positive since $Q^{\mathrm{III}}(\theta)\geq 0$ for all $\theta\geq\theta^{0}$.

Next, we substitute (3) into (6) and (18) into (19) and rearrange terms to get

$$W^{\mathrm{III}}-W^{\mathrm{I}}=\int_{\theta^{\mathrm{I}}}^{1}\frac{(\theta\mathrm{R}-\mathrm{r})^2}{2\gamma r^2}d\theta+\int_{\theta^0}^{\theta^{\mathrm{I}}}\frac{(\theta\alpha-\mathrm{r})^2}{2\gamma r^2}d\theta>0.$$

Q.E.D.

***Proof of Proposition* 3**: In this proof, we drop superscript IV if no confusion arises, and use subscripts to denote partial derivatives. To show the existence of the signalling equilibrium, we follow the standard approach of verifying Riley's six conditions. We first define $\varphi(\theta; Q)$ as the true success probability of a firm of type θ, which is a function of θ and (possibly) Q. Note that $\varphi(\theta; Q) \equiv \theta$ in our model. We restate Riley's six conditions for our model as follows:

(R1) The unobservable credit quality θ is distributed on the closed interval [0, 1] according to the strictly increasing function $F(\theta) \in C^{\infty}$;

(R2) $V(\theta; Q, P)$ in (21) and $\varphi(\theta; Q)$ are differentiable in all their arguments;

(R3) $V_P(\theta; Q, P) > 0$;

(R4) $\varphi(\theta; Q) > 0$, $\varphi_\theta(\theta; Q) > 0$, and $\varphi_Q(\theta; Q) \geq 0$;

(R5) $\frac{\partial}{\partial \theta}\left(-\frac{V_Q}{V_P}\right) < 0$; and

(R6) $\forall \theta \in [0, 1]$, $V(\theta; Q, \varphi(\theta; Q))$ has a unique turning point at $Q^{\text{III}}(\theta)$, i.e. the value maximizing level of investment. Moreover, $\forall \theta \in [0, 1]$ it holds that $V(\theta; \infty, \varphi(\theta, \infty)) < V(\theta; Q^{\text{III}}(0), \varphi(0; Q^{\text{III}}(0)))$.

We check these conditions as follows. (R1) is satisfied by the uniform distribution assumption. (R2) easily follows from V in (21) and from the definition of $\varphi(\theta; Q)$. To see that (R3) holds, we differentiate V in (21) partially with respect to P to get

$$V_P = \frac{\theta Q}{P^2} > 0. \tag{A6}$$

(R4) is satisfied since $\varphi(\theta; Q) \equiv \theta$. Note that firms of type $\theta = 0$ can be ignored in our analysis. For the sake of (R5), we first differentiate V in (21) partially with respect to Q to get

$$V_Q = \frac{\theta(\alpha - r/P)}{r} - \gamma Q. \tag{A7}$$

Using (A6) and (A7), we get the marginal rate of substitution between P and Q as

$$\frac{\partial}{\partial \theta}\left(-\frac{V_Q}{V_P}\right) = -\frac{\gamma P^2}{\theta^2} < 0 \quad \text{for all } \theta.$$

Finally, the first part of (R6) is satisfied because $Q^{\text{III}}(\theta)$ in (17) uniquely maximizes $V(\theta; Q, \varphi(\theta, Q))$ since $\varphi(\theta, Q) \equiv \theta$. The second part is also satisfied because $V(\theta;\ \infty,\ \varphi(\theta;\ \infty))=-\infty$ and $V(\theta;\ Q^o(\theta^o),\ \varphi(\theta;\ Q^o(\theta^o)))=0$ since $Q^{III}(\theta)=0$ for all $\theta \leq \theta^o$.

Now, we show the two properties of the equilibrium. (i) In (21), we substitute $\varphi(\theta;\ Q)\equiv\theta$ into $P(Q)$ to write

$$V=\frac{(\theta\alpha-r)Q}{r}-\frac{\gamma Q^2}{2}. \tag{A8}$$

For $\theta < \theta^o$, the value in (A8) becomes nonpositive for $Q \geq 0$. Therefore, we obtain $Q^{IV}(\theta)=0$. For $\theta \geq \theta^o$, on the other hand, $Q^{IV}(\theta)$ must satisfy the following first order condition:

$$V_Q+V_P\frac{dP}{dQ}=0. \tag{A9}$$

Applying the implicit function theorem to this equation, we get

$$\frac{dQ^{IV}}{d\theta}=-\frac{V_{Q\theta}+V_{P\theta}\dfrac{dP}{dQ}}{\dfrac{d^2V}{dQ^2}}>0 \quad \text{for all } \theta \geq \theta^o. \tag{A10}$$

We obtain the positive sign in (A10) as follows: The denominator is negative by the strict concavity of $V(\theta;\ Q,\ \varphi(\theta;\ Q))$ in Q given by (R6). One can alternatively check this by using (21) and $\varphi(\theta,\ Q)\equiv\theta$. By using (A6), (A7) and (A9), we rewrite the numerator as

$$\frac{\alpha-r/P}{r}+\frac{Q}{P^2}\left(-\frac{V_Q}{V_P}\right)=\frac{\gamma Q}{\theta}>0.$$

(ii) From (22), we have $(dP/dQ)\cdot(dQ^{IV}/d\theta)=1$. Thus, it follows from (A10) that $dP/dQ>0$. The desired result is then immediate from (20).

Q.E.D.

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