Contests with Externalities

Sanghack Lee [1]

This paper examines contests in which aggregate efforts generate positive externalities to participants. In such contests the equilibrium effort may exceed or fall short of the socially optimal level of effort. This paper derives the relationship between the equilibrium effort, the size of prize, and the socially optimal level of effort. The equilibrium effort proves to exceed the social optimum when it is less than the prize times the exponent R of the Tullock (1980) contest-success function. On the other hand, when the equilibrium effort is greater than the prize times the exponent R, it indeed falls short of the social optimum.

Keywords: Contest, Externality, Social optimum

JEL Classification: D62, D71, D72

I. Introduction

There exists a large set of economic, political and social interactions that can be modeled as contests. In such contests players compete to win prizes such as fame, government subsidies, monopoly rents, patents, prestige, promotion, and so on. Many authors have examined characteristics and efficiency implications of contests in various settings. Examples include Tullock (1980), Appelbaum and Katz (1987), Cleeton (1989), Lee (1993, 1995), and Mantell (1996).[1]

*Professor, Division of International Trade, School of Economics, Kookmin University, Seoul 136-702, Korea, (Tel) +82-2-910-4546, (Fax) +82-2-910-4519, (E-mail) slee@kmu.kookmin.ac.kr. I wish to thank K. H. Baik, J. H. Kang and two anonymous referees for their valuable comments and suggestions on an earlier version. Financial support from Kookmin University is gratefully acknowledged.

[1]For a survey of the literature, see Nitzan (1994).

[**Seoul Journal of Economics** 2000, Vol. 13, No. 2]

Aggregate efforts made in contests often generate positive externalities to contest participants. Reflecting this, several authors have examined implications of the externalities on contest outcomes and social welfare. Congleton (1989) has analyzed status-seeking contests in which individuals not actively involved in the contests are affected by externalities. Chung (1996) has analyzed a rent-seeking contest where the size of the rent increases in aggregate efforts. Lee and Kang (1998) have examined contests in which the aggregate efforts exhibit characteristics of a pure public good. Baik and Lee (2000) have examined a two-stage contest with spillover effects between the stages.

Contests with externalities are indeed ubiquitous. To facilitate intuitive understanding, several examples of contests with externalities are now offered. The first example is a cost-reducing R&D contest between firms in which the aggregate expenditures on R&D have cost-reducing spillover effects on all firms, (see, *e.g.* d'Aspremont and Jacquemin 1988; and Suzumura 1992). One of the firms can obtain a patent as a result of the R&D contest. This patent is the prize of the contest, while the cost-reducing effect is captured by the externality term. Both the winner and the losers can obtain positive spillover effect from the aggregate expenditures on R&D. Therefore, expenditures of the losing firms are not totally wasted. Even if there is no cost-reducing effect, some proportion of total outlays on R&D contest can be recovered in the form of increased R&D know-how, training of researchers, and spillover of knowledge between the firms.

Sports contests are also likely to be associated with externalities. Most participants, for example, though not all, in Olympic games rejoice in feeling of pride or accomplishment regardless of the outcome of the contest. Art and music contests also generate positive externalities to participants, for they obtain prestige, pride, and experience, while expending resources to win prizes. The level of pride of the participants in these contests is certainly related to the level of aggregate efforts made in the contests.

Competition for publication in academic journals is an example pertaining to the academic profession. The more manuscripts are submitted to journals, the higher the average quality of published papers. The authors of rejected papers would also obtain benefits from referees' comments and editors' general guidance, since they can improve analytical and stylish skill through the refereeing process. Improvements in quality of published papers and wide dissemination of knowledge through this process are positive externalities associated with competition for publication.

The purpose of this paper is to examine contests in which aggregate efforts generate positive externalities with characteristics of a club-good. Extending the work of Chung (1996) and Lee and Kang (1998), this paper allows for the possibility of congestion in externalities. Section II sets out the model of the contest associated with externalities. This paper also compares the equilibrium effort with the socially optimal level of effort. With externalities in the contests, the aggregate effort may exceed or fall short of the socially optimal level of effort. This paper offers a simple rule by which one can tell whether the effort made in the contests is excessive or not. The equilibrium effort proves to exceed the social optimum when it is less than the prize times the exponent R of the contest-success function adopted by Tullock (1980). On the other hand, when the equilibrium effort is greater than the prize times the exponent R, it indeed falls short of the social optimum. The comparative static effects of changes in prizes and number of players are also examined. Concluding remarks are offered in Section III.

II. The Model

Consider a contest in which n risk-neutral players compete to win a private-good prize. The size of prize is S. To win the prize, player i expends x_i in units commensurate with the prize. As in the literature following Tullock (1980), player i's probability of winning, Π_i, is given by a logit-form function

$$\Pi_i = \frac{(x_i)^R}{\sum_j (x_j)^R}, \tag{1}$$

where the exponent $R(>0)$ represents the degree of marginal returns to efforts. The aggregate outlays are denoted by $X(=\sum_j x_j)$.

The contest is assumed to be associated with externalities in the sense that each player's real cost of participation in the contest is affected by the aggregate outlays[2] (This follows the literature on R&D contest, *e.g.* d'Aspremont and Jacquemin 1988; and Suzumura

1992). Specifically, player i's real cost is assumed to be given by $(x_i - f(X)/n^r)$, where $f(X)/n^r$ satisfies the following assumptions:

(A1) $f(X)$ is twice continuously differentiable.

(A2) $f(0)=0$, $f'(X)\geq 0$ and $f''(X)\leq 0$, where the equality sign in the second weak inequality holds only when $f'(0)=0$.

(A3) $r\geq 0$.

(A4) For any finite and positive S, there exists a finite, positive X^b such that $S+f(X^b)/n^{r-1}=X^b$.

The meaning of each assumption is transparent. The larger the aggregate effort, the greater is $f(X)$. The denominator n^r captures congestion effects. The larger r is, the greater the congestion effect, and when $r=0$, there is no congestion effect at all. If $r=1$, the aggregate externality is independent of the number of participants in the contest. (A4) is adopted to guarantee existence of a finite social optimum. Note that this model is reduced to the conventional rent-seeking contest when $f(X)=0$. The model of Lee and Kang (1998), where $f(X)$ is given by βX, satisfies all of the above assumptions except the last part of (A2), that of $f''(X)<0$.

The expected payoff of participation in the contest to player i, V_i, is given by

$$V_i = \Pi_i\left\{ S-\left(x_i-\frac{f(X)}{n^r}\right)\right\}+(1-\Pi_i)\left\{-\left(x_i-\frac{f(X)}{n^r}\right)\right\} \tag{2a}$$

$$=\Pi_i(S-x_i)+(1-\Pi_i)(-x_i)+\frac{f(X)}{n^r} \tag{2b}$$

$$=S\Pi_i-x_i+\frac{f(X)}{n^r}. \tag{2c}$$

To make explicit two possible interpretations of externality, the payoff function V_i is written in two different forms as equations (2a) and (2b). Equation (2a) corresponds to the above interpretation that each player's real cost is reduced due to externality effects. Equation (2b) represents the other interpretation that each player obtains some proportion of aggregate efforts as an externality effect,

[2] It is quite possible that non-participants are also affected by the aggregate effort, as in Congleton (1989). The present paper abstracts from this possibility.

irrespective of whether he is a winner or not. This interpretation seems valid in art, music, or athletic contests. Note that both (2a) and (2b) yield the same form of payoff function as equation (2c). With slight modification, this contest can also be viewed as one in which the size of the prize increases in aggregate effort, (see Chung (1996)). In this case, the sum of the prize and the aggregate externality is given by $S+f(X)/n^{r-1}$, which accrues solely to the winner. However, in the present model the increased portion $f(X)/n^{r-1}$ is equally shared by all the players.

The sum of individual payoffs is $S-X+f(X)/n^{r-1}$. This is maximized at the socially optimal level of efforts X^*. Note that the social optimum X^* is independent of S. When $f'(0)>n^{r-1}$, there exists a unique and positive social optimum X^*. In this case, it also follows that $f'(X^*)=n^{r-1}$. On the other hand, if the externalities are not large enough so that $f'(0)\leq n^{r-1}$, then the social optimum is given by $X^*=0$. In the conventional rent-seeking contest, the social optimum X^* is zero, since $f'(X)=0$. When $X^*>0$, by implicit function theorem

$$\frac{dX^*}{dn}=\frac{(r-1)n^{r-2}}{f''(X^*)}. \tag{3}$$

When $r>1$, $dX^*/dn<0$ since $f''(X)<0$. This is the case where serious congestion is associated with the externality. Therefore an increase in the number of participants decreases the socially optimal level of effort. When $r<1$, the congestion is not large enough to offset the positive externality resulting from an increase in n. Thus, it follows that $dX^*/dn>0$.[3]

The Nash equilibrium effort is now derived. The objective function of risk-neutral player i is:

$$\underset{x_i}{\text{Max}}\ V_i=S\Pi_i-x_i+\frac{f(X)}{n^r}.$$

[3]Caution is required when interpreting this result. When n is small, X^* can be zero. For a sufficiently large n, X^* is positive. For a small decrease in n, there can be a jump of X^* from positive value to zero. Conversely, X^* can jump from zero to a positive value for a small increase in n, when n is small.

Each player decides the level of his or her effort, taking all the other players' decisions as given. Then, the first-order condition for the above maximization problem is

$$\frac{\partial V_i}{\partial x_i}=\frac{SR[(x_i)^{R-1}(\sum_j (x_j)^R)-(x_i)^{2R-1}]}{(\sum_j (x_j)^R)^2}-1+\frac{f'(X)}{n^r}=0, \quad \text{for } i=1,\cdots,n. \quad (4)$$

Assuming symmetry, equation (4) is simplified as

$$\frac{(n-1)RS}{nX}-1+\frac{f'(X)}{n^r}=0. \quad (5)$$

To simplify the exposition, let us define a function of aggregate effort

$$g(X)\equiv(n-1)RS-nX+\frac{Xf'(X)}{n^{r-1}}.$$

Then, at the Nash equilibrium X^N, $g(X)$ has a value of zero, *i.e.* $g(X^N)=0$.

The Nash equilibrium X^N is now compared with the social optimum X^* in four cases which are mutually exclusive and exhaustive of all possibilities. When $f'(0)\leq n^{r-1}$, the externality effect is not large enough and the social optimum is given by $X^*=0$, (Case 1). When $f'(0)>n^{r-1}$, the externality effect is strong enough to ensure a positive social optimum. There are three distinctive cases depending upon the size of X^* relative to the prize times the exponent R, RS, (Cases 2, 3 and 4).[4]

***Case* 1:** $f'(0)\leq n^{r-1}$

In this case the externality effect is not large enough to warrant a positive social optimum so that $X^*=0$. Note that $g(0)=(n-1)RS>0$. Also,

$$g(RS)=RS\left\{\frac{f'(RS)}{n^{r-1}}-1\right\}<RS\left\{\frac{f'(0)}{n^{r-1}}-1\right\}\leq 0,$$

[4]Note that X^* is independent of S.

since $f'(RS)/n^{r-1} < f'(0)/n^{r-1}$ by the assumption that $f''(X) < 0$ and $f'(0) \leq n^{r-1}$. For any $X \geq 0$,

$$g'(X) = \left\{\frac{f'(X)}{n^{r-1}} - 1\right\} + \frac{Xf''(X)}{n^{r-1}} - (n-1) < 0.$$

Thus, the equation $g(X) = 0$ has a unique solution X^N that belongs to the interval $(0, RS)$. That is, $X^* = 0 < X^N < RS$. Social waste results from the contest since $X^N > f(X^N)/n^{r-1}$. Note that this case encompasses most imperfectly discriminating contests analyzed in the rent-seeking literature.

***Case* 2:** $n^{r-1} < f'(0)$ and $X^* < RS$

This is the case where the externality is strong enough to ensure a positive social optimum of efforts, and where the social optimum is less than R times the prize. $g(X)$ is now rewritten as

$$g(X) = (n-1)(RS - X) + X\left\{\frac{f'(X)}{n^{r-1}} - 1\right\}.$$

Then it is easy to find that

$$g(RS) = RS\left\{\frac{f'(RS)}{n^{r-1}} - 1\right\} < RS\left\{\frac{f'(X^*)}{n^{r-1}} - 1\right\} = 0,$$

since $f'(RS) < f'(X^*)$. Also,

$$g(X^*) = (n-1)(RS - X^*) + X^*\left\{\frac{f'(X^*)}{n^{r-1}} - 1\right\} = (n-1)(RS - X^*) > 0.$$

For any $X < X^*$,

$$g(X) = (n-1)(RS - X) + X\left\{\frac{f'(X)}{n^{r-1}} - 1\right\} > 0,$$

since $RS > X^* > X$ and $(f'(X)/n^{r-1} - 1) > 0$. For any $X \geq X^*$,

$$g'(X)=\frac{Xf''(X)}{n^{r-1}}+\left\{\frac{f'(X)}{n^{r-1}}-1\right\}-(n-1)<0.$$

These results in sum indicate that there exists a unique X^N between X^* and RS, *i.e.* $X^*<X^N<RS$.

***Case* 3:** $n^{r-1}<f'(0)$ and $RS<X^*$
In this case $g(RS)=RS(f'(RS)/n^{r-1}-1)>0$. For any $X<RS$, $g(X)>0$. Also, $g(X^*)=(n-1)(RS-X^*)<0$. For any $X\geq X^*$,

$$g'(X)=\frac{Xf''(X)}{n^{r-1}}+\left\{\frac{f'(X)}{n^{r-1}}-1\right\}-(n-1)<0.$$

It thus follows that X^N lies between RS and X^*, *i.e.* $RS<X^N<X^*$. In the neighborhood of X^N, it also follows that

$$g'(X)=\frac{Xf''(X)}{n^{r-1}}+\frac{f'(X)}{n^{r-1}}-n=\frac{Xf''(X)}{n^{r-1}}-\frac{(n-1)RS}{X}<0 \quad \text{(by equation (5))}.$$

It can be shown that the equilibrium is unique. The contest generates social surplus since $X^N<f(X^N)/n^{r-1}$. However, the surplus is not maximized since X^N falls short of the social optimum X^*.

***Case* 4:** $n^{r-1}<f'(0)$ and $RS=X^*$
For any $X<X^*(=RS)$,

$$g(X)=(n-1)(RS-X)+X\left\{\frac{f'(X)}{n^{r-1}}-1\right\}>0.$$

Also, for any $X>X^*(=RS)$,

$$g(X)=(n-1)(RS-X)+X\left\{\frac{f'(X)}{n^{r-1}}-1\right\}<0.$$

In addition, $g(X^*)=0$. Therefore it follows that $RS=X^N=X^*$. Surplus from the contest, $f(X)/n^{r-1}-X$, is maximized.

From the above analysis of the four cases, we derive the important relationship between the social optimum and the Nash equilibrium effort. In Cases 1 and 2, $X^* < X^N < RS$. In Case 3, $RS < X^N < X^*$. X^N always lies between X^* and RS. Also, $X^* = X^N = RS$ in Case 4. These results are succinctly summarized as proposition 1.

***Proposition* 1**

The set of X^*, X^N and RS satisfies one of the following three conditions:

(1) $X^* < X^N < RS$, (2) $X^* = X^N = RS$, or (3) $X^* > X^N > RS$.

When $X^N < RS$, the efforts of participants in the contest might seem lukewarm. However, the equilibrium efforts exceed the social optimum. The rent-seeking contests belong to this case. On the other hand, when $X^N > RS$, one might be impressed that too much effort is being made in the contest. However, the equilibrium effort falls short of the social optimum. If the aggregate effort is observable, then one can obtain definite welfare implications of the contest. The rent-seeking literature has focused on the case where $R=1$. In this case the simple comparison of X^N with S yields definite welfare implications: Under-dissipation of the rent indicates that social costs are incurred in the rent-seeking contest.

This paper now derives the comparative static effects of changes in the size of prize and the number of participants. These results might be utilized when making the equilibrium expenditure converge to the social optimum by adjusting the size of prize and number of contenders. Total differentiation and arrangement of equation (5) gives

$$\frac{dX^N}{dS} = \frac{-R(n-1)}{\dfrac{f'(X)}{n^{r-1}} + \dfrac{Xf''(X)}{n^{r-1}} - n} = \frac{-R(n-1)}{\dfrac{Xf''(X)}{n^{r-1}} - \dfrac{(n-1)RS}{X}}, \tag{6}$$

and

$$\frac{dX^N}{dn} = -\frac{\{(r+1)n^r - rn^{r-1}\}RS - (r+1)n^r X + Xf'(X)}{-n^{r+1} + nf'(X) + nXf''(X)}. \tag{7}$$

From equation (5) it is easy to find out that $f'(X)/n^{r-1} = \{n-(n-1)RS/X\}$. This can be used to show that the denominator of (6) is

$$\frac{f'(X)}{n^{r-1}}+\frac{Xf''(X)}{n^{r-1}}-n=\frac{Xf''(X)}{n^{r-1}}-\frac{(n-1)RS}{X}<0.$$

Thus, $dX^N/dS>0$. When the equilibrium effort falls short of R times the prize, a reduction in the prize is suggested. This would reduce the equilibrium effort, thereby making it converge to the social optimum, and *vice versa*.

Equation (7) is now examined to see the effect of change in the number of participants. Again from equation (5), it can be shown that $1>f'(X^N)/n^r$. This implies that denominator of equation (7) is

$$-n^{r+1}+nf'(X)+nXf''(X)=-n^{r+1}(1-\frac{f'(X)}{n^r})+nXf''(X)<0.$$

Thus, the sign of (dX^N/dn) is the same as the sign of numerator of equation (7). Simple manipulation shows that $\text{sign}(dX^N/dn)=\text{sign}[(rn-r+1)RS-rnX]$. If $X^*<X^N<RS$ and $r\leq 1$, then $dX^N/dn>0$. If $RS<X^N<X^*$ and $r\geq 1$, then $dX^N/dn<0$. These results are summarized as proposition 2.

Proposition 2

(1) $dX^N/dS>0$.

(2) The sign of dX^N/dn is indeterminate.

III. Concluding Remarks

This paper has examined contests associated with externalities. It is shown that the equilibrium effort may exceed or fall short of the social optimum depending upon the extent of externalities. This paper has derived the rule by which one can tell whether the aggregate effort made in the contest fall short of, or exceed the social optimum. The critical level turns out to be the prize times the exponent R of the Tullock (1980) contest-success function. When the aggregate effort falls short of the critical value, then it indeed exceeds the social optimum. When the aggregate effort exceeds the critical level, one might be impressed that excessive effort is being made in the contest. However, it falls short of the social optimum. An adjustment in the prize or in the number of participants can induce changes in the equilibrium effort. While the change in the prize has unambiguous effect on the equilibrium effort, the effect of a change in the number of participants is ambiguous.

(*Received January, 2000; Revised March, 2000*)

References

Appelbaum, E., and Katz, E. "Seeking Rents by Setting Rents: The Political Economy of Rent Seeking." *Economic Journal* 97 (1987): 685-99.

Baik, K. H., and Lee, S. "Two-Stage Rent-Seeking Contests with Carryovers." *Public Choice* 103 (Nos. 3-4 2000): 285-96.

Chung, T. Y. "Rent-seeking Contests When the Prize Increases with Aggregate Efforts." *Public Choice* 87 (No. 1 1996): 55-66.

Cleeton, D. L. "Equilibrium Conditions for Efficient Rent-Seeking: The Nash-Cournot Solution." *Quarterly Review of Economics and Business* 29 (No. 2 1989): 6-14.

Congleton, R. "Efficient Status Seeking: Externalities, and the Evolution of Status Games." *Journal of Economic Behavior and Organization* 11 (1989): 175-90.

D'Aspremont, C., and Jacquemin, A. "Cooperative and Noncooperative R&D in Duopoly with Spillovers." *American Economic Review* 78 (1988): 1133-7.

Lee, S. "Inter-Group Competition for a Pure Private Rent." *Quarterly Review of Economics and Finance* 33 (No. 3 1993): 261-6.

__________. "Endogenous Sharing Rules in Collective-Group Rent-Seeking." *Public Choice* 85 (Nos. 1-2 1995): 31-44.

Lee, S., and Kang, J. H. "Collective Contests with Externalities." *European Journal of Political Economy* 14 (No. 4 1998): 727-38.

Mantell, E. "The Social Costs of Monopoly and Regulation: Posner Reconsidered Again." *Quarterly Review of Economics and Finance* 36 (1996): 249-68.

Nitzan, S. "Modelling Rent-Seeking Contests." *European Journal of Political Economy* 10 (No. 1 1994): 41-60.

Suzumura, K. "Cooperative and Noncooperative R&D in an Oligopoly with Spillovers." *American Economic Review* 82 (1992): 1307-20.

Tullock, G. "Efficient Rent Seeking." In J. Buchanan, R. Tollison, and G. Tullock (eds.), *Towards a Theory of the Rent-Seeking Society*. College Station: Texas A&M University Press, 1980.