Optimal Contracts for Risk Managers

Jin Yong Jung

This study analyzed the principal-agent problem, in which the agent performs risk management tasks, and considered the cost minimization problem of the principal, the objective of which is to design the cheapest contract inducing a target effort. Our results confirm that a one-step bonus contract should be used, which means that a bonus contract is most efficient for the principal in terms of incentive provision. A new condition to justify the first-order approach in our model was also provided.

Keywords: Risk managers, Risk-reducing effort, Bonus contract

JEL Classification: D82, D86, G32, J33, J41

I. Introduction

After the recent financial crisis, the importance of risk management has been emphasized among many firms (*e.g.,* financial companies) because the failure of risk management is widely recognized as its main cause. Among the various causes of the risk management failure, Flaherty *et al.* (2013) elucidated that misguided compensation structures enhanced risk-taking incentives. Hence, risk managers were compensated with incentive contracts to motivate their selection of risky projects rather than safe ones.

Many contracts, which are typical in real firms, provide incentives to increase firm risk for their top managers. For instance, option-based

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compensations for chief executive officers (CEOs) create incentives to select risky projects by providing convex payoff structures, thereby increasing firm risk.[1] Tournament contracts also have a similar effect. Intra-organizational CEO promotion tournaments provide senior executives below the CEO position with risk-taking incentives, which increases firm risk.[2]

The current study aims to determine the optimal contracts for risk managers. Therefore, we consider the principal-agent problem wherein an agent plays the role of risk manager on behalf of a principal (firm). The effort of the agent considered in our model induces a different effect compared with classical principal-agent models. In classical agency models, the agent's effort improves the return of the principal in the sense of first-order stochastic dominance. Thus, the monotone likelihood ratio property (MLRP) is assumed.[3] However, the effort in our model reduces the risk of return. Thus, we cannot assume the MLRP.

Palomino and Prat (2003) examined the moral hazard problem where the agent can select the risk of return similar to our model. They considered the principal-agent problem wherein the agent has a limited liability and can sabotage or destroy the realized outcome.[4] In their model, the agent assumes the role of money manager, and can gather information on investment opportunities by exerting effort. Thus, the effort of the agent increases the mean of return and its risk simultaneously by expanding a set of his feasible portfolios.[5] Palomino and Prat (2003) corroborated that a one-step bonus contract is one of optimal contracts for money managers, under the assumption that the agent can select between low or high effort levels.

We also consider the principal-agent problem wherein the agent has

[1] See Smith and Stulz (1985); Guay (1999); Coles *et al.* (2006); Chava and Purnanandam (2010).

[2] See Kini and Williams (2012).

[3] Note that MLRP implies the first-order stochastic dominance, which means that great outcomes are evidence of great effort by the agent. For a detailed explanation, see Milgrom (1981).

[4] The optimal contract must be a non-decreasing function because of the sabotaging ability of the agent. For a detailed explanation, see Innes (1990).

[5] More generally, the agent is considered to be able to increase the expected return and select its risk, separately. For such models, refer to Hirshleifer and Suh (1992), Sung (1995), and Kim (2005) among others.

a limited liability and can sabotage the realized outcome. However, in our model, the agent can reduce the risk of return directly by exerting effort. Thus, we assume the likelihood ratio increases from negative and decreases to negative as the return increases. This assumption means more effort from the agent reduces the risk of return in the sense of mean preserving spread.

We analyze the cost minimization problem of the principal containing the incentive compatibility and the individual rationality constraints. The individual rationality constraint may not be binding at the optimum because of the agent's limited liability, which makes our problem difficult to handle. Instead, we first deal with the relaxed cost minimization problem wherein the individual rationality constraint is removed and analyze the original problem.

We show that a one-step bonus contract is optimal for solving the relaxed cost minimization problem, which means that inducing any given effort in this bonus contract is cheap. Thus, a one-step bonus contract is efficient for the principal in terms of the incentive provision for the agent assuming the risk manager role.

This result also implies directly that, if the bonus contract to solve the relaxed problem satisfies even the individual rationality constraint of the agent, such a bonus contract can solve the original cost minimization problem. Because the agent will naturally accept an efficient bonus contract in terms of incentive provision. However, if such a bonus contract does not satisfy the individual rationality constraint, then it cannot solve the original cost minimization problem because its expectation is considerably low to be accepted by the agent. Thus, a new bonus contract obtained by adding a positive constant can solve the original problem, making the individual rationality constraint satisfied at a minimum.

It is worth noting that the optimality of one-step bonus contract pertains more to risk neutral agent and to increasing pay schedule rather than the shape of the likelihood ratio. According to Kim (1997), the one-step bonus contract can be an optimal contract for the risk neutral agent even under the MLRP. Thus, the optimality of bonus contract depends on risk neutrality of the agent and increasing pay schedule. Nevertheless, the uniqueness of bonus contract depends critically on the likelihood ratio's shape assumed in this study.

This paper is organized as follows. Section II suggests our basic model; Section III provides our results; Section IV deals with the issue of the first-order approach; and Section V concludes this paper.

II. Basic Model

We consider the principal-agent problem wherein an agent assumes the role of managing firm risk. The firm (or principal) working on certain projects with the high risks and returns concerns high risks inherent in them. For the principal that allows an agent to manage and reduce risks, the agent's effort negatively affects firm risk.

The principal and the agent are risk neutral. Upon employment, the agent exerts his effort $a \in A \equiv [0, \bar{a}]$ with cost $c(a)$ to reduce the risk of return $\tilde{x}$. $c(a)$ denotes the effort cost of the agent with $c'(a) > 0$ and $c''(a) \geq 0$. Subsequently, the return, which is affected stochastically by effort choice a of the agent, is realized. The principal cannot observe effort choice a of the agent, but realized return x is commonly observable without costs. Thus, wage contracts for the agent should be designed depending on the return x, *i.e.*, $w = w(x)$.

We assume that the agent has a limited liability. Thereafter, wage contract $w(x)$ must not always be less than zero [*i.e.*, $w(x) \geq 0$ for all $x \in X = (\underline{x}, \bar{x})$], where X is the support of random variable $\tilde{x}$. The agent can sabotage or destroy realized return x according to Innes (1990). This assumption requires optimal contracts for the agent to be non-decreasing in x.

Given that the distribution of return $\tilde{x}$ depends on the effort a of the agent, its cumulative distribution function is denoted by $F(x|a)$, and the corresponding probability density function is denoted by $f(x|a)$. $F(x|a)$ and $f(x|a)$ are twice differentiable. Moreover, support $X = (\underline{x}, \bar{x})$ is independent of a. Hence, its lower bound $\underline{x}$ and upper bound $\bar{x}$ are irrelevant to the effort a of the agent.

The effort a of the agent has a negative effect on the risk of return $\tilde{x}$. Therefore, we assume that, for any a, as x increases, the likelihood ratio $f_a(x|a)/f(x|a)$ increases from negative and decreases to negative.[6] This assumption, under the condition that $E[x|a]$ is independent of a, implies that an increase in effort a reduces the risk of return $\tilde{x}$.

***Lemma* 1**. Suppose that $\mu(a) \equiv E[x|a]$ is independent of a. Subsequently,

[6] Hence, we do not assume the MLRP.

for any a, $\int^{x} F_a(t|a)dt \le 0$ for all $x \in X$.

Proof. Given that $E[f_a(x|a)/f(x|a)] = 0$ because $\int f_a(x|a)dx = 0$, the likelihood ratio $f_a(x|a)/f(x|a)$ for any a should go through x-axis at least once. Combining it with the assumption that $f_a(x|a)/f(x|a)$ increases from negative and then decreases to negative implies that, for any a, the sign of $f_a(x|a)/f(x|a)$ changes from negative to positive and then to negative as x increases. Given that the sign of $f_a(x|a)$ has the same pattern with $f_a(x|a)/f(x|a)$, we have $f_a(x|a) > 0$ for all $x \in (x_1, x_2)$, but $f_a(x|a) < 0$ for all $x \in (\underline{x}, x_1) \cup (x_2, \bar{x})$, where x_1 and x_2 are the first and the second x-intercepts of $f_a(x|a)$, respectively. Given that $F_a(x|a) = \int^{x} f_a(t|a)dt$, $F_a(x|a)$ is increasing on interval (x_1, x_2) but decreasing on both intervals $(\underline{x}, x_1)$ and $(x_2, \bar{x})$. Moreover, because the lower bound $\underline{x}$ and upper bound $\bar{x}$ of the support are independent of a, $F(\underline{x}|a) = 0$ and $F(\bar{x}|a) = 1$ for all a, which implies that $F_a(\underline{x}|a) = F_a(\bar{x}|a) = 0$. Thus, the sign of $F_a(x|a)$ changes from negative to positive as x increases.

Let $\hat{F}(x|a) = \int^{x} F(t|a)dt$. Note that $\hat{F}_a(\underline{x}|a) = 0$ since $\hat{F}(\underline{x}|a) = 0$ for all a. In addition, $\mu(a) \equiv \int x f(x|a)dx = \underline{x} + \int[1 - F(x|a)]dx$, where the second equality can be derived by using integration by parts. Given that $\mu(a)$ is independent of a, we have $\mu'(a) = -\int F_a(x|a)dx = 0$, which is equivalent to $\hat{F}_a(\bar{x}|a) = 0$.

For any a, considering that the sign of $F_a(x|a)$ changes from negative to positive once as x increases, we have $F_a(x|a) < 0$ for all $x \in (\underline{x}, x_3)$, but $F_a(x|a) > 0$ for all $x \in (x_3, \bar{x})$, where x_3 denotes the x-intercept of $F_a(x|a)$, which means $\hat{F}_a(x|a) = \int^{x} F_a(t|a)dt$ is decreasing on interval $(\underline{x}, x_3)$ but increasing on interval $(x_3, \bar{x})$. This scenario, together with $\hat{F}_a(\underline{x}|a) = 0$ and $\hat{F}_a(\bar{x}|a) = 0$, implies that $\hat{F}_a(x|a)$ is always negative, or equivalently, $\int^{x} F_a(t|a)dt \le 0$, for all x and a. Q.E.D.

The above lemma shows that, under the condition that the effort of the agent does not affect the expected return at all, our assumption regarding the likelihood ratio implies that, for any $a_2 > a_1$, $\int^{x} F(t|a_1)dt \ge \int^{x} F(t|a_2)dt$, for all x, thereby indicating that distribution $F(x|a_1)$ under low effort a_1 is a mean preserving spread of distribution $F(x|a_2)$ under high effort a_2 and implies that the increase in the effort of the agent reduces the risk of return $\tilde{x}$.[7]

[7] This lemma still holds even in the case where $\mu(a)$ is increasing in a. As shown in the proof of Lemma 1, $\hat{F}_a(\bar{x}|a) \le 0$ is trivial, which makes the proof still valid. In this case, Lemma 1 indicates the effort of the agent improves the

Our assumption regarding the likelihood ratio can be satisfied in many distributions only if the effort of the agent plays a role in reducing the risk regardless of whether the expected return is constant in his effort. This possibility shows our assumption is not profoundly restrictive and the effect of the agent's effort on the expected return is not associated with our assumption. For instance, consider the Normal distribution with mean $\mu(a) > 0$ and variance $\sigma^2(a) > 0$, denoted by $\tilde{x} \sim N(\mu(a), \sigma^2(a))$. In this case, the likelihood ratio is

$$\frac{f_a(x \mid a)}{f(x \mid a)} = \frac{[x - \mu(a)]^2}{\sigma^3(a)} \sigma'(a) + \frac{x - \mu(a)}{\sigma^2(a)} \mu'(a) - \frac{\sigma'(a)}{\sigma(a)}.$$

In this equation, if $\sigma(a)$ is decreasing in a (*i.e.*, $\sigma'(a) < 0$), then our assumption is satisfied regardless of the sign of $\mu'(a)$.

Moreover, the likelihood ratio under our assumption measures the variability of return, which indicates that the optimal contract may depend on the variability of return, implying that the optimal contract should increase and then decrease as x increases. However, when the agent can sabotage the realized outcome x, such a contract does not work. Specifically, when the outcome is expected to be realized on a decreasing part of that contract, the agent receives incentives to destroy part of the outcome and report a performance lower than actual one. Therefore, when the agent has a sabotaging activity, his contract cannot depend on variability.

We consider the cost minimization problem of the principal to induce any given effort $a > 0$. The problem of the principal is represented as follows:

Original CMP:

$$C(a) \equiv \min_{w(x) \in W} \int w(x) f(x \mid a) dx,$$

distribution of return $F(x|a)$ in the sense of second-order stochastic dominance. Nevertheless, Lemma 1 is proven in the case that $\mu(a)$ is independent of a, emphasizing that our assumption on the likelihood ratio sufficiently supports that the effort of the agent decreases the risk of return $\tilde{x}$ in the sense of mean preserving spread.

subject to

$$\int w(x) f_a(x|a)dx - c'(a) = 0, \qquad \text{(IC)},$$
$$\int w(x) f(x|a)dx - c(a) \geq \bar{U}, \qquad \text{(IR)},$$

where $W \equiv \{w(x)|w(x) \geq 0,\ w'(x) \geq 0\}$ is the feasible set for the principal constrained by limited liability and sabotaging activity of the agent. In the original CMP, the first constraint is called the incentive compatibility constraint of the agent to adopt the first-order approach. The second is called his individual rationality constraint, where $\bar{U}$ denotes the reservation utility level of the agent.

If (IR) must be binding at the optimum, optimal contracts used to solve the original CMP are any contract to make (IR) and (IC) binding. Thus, many optimal contracts may exist in this case. However, because the agent has a limited liability, (IR) may not be binding at the optimum. Thus, finding the optimal contract to solve the original CMP is not easy. Furthermore, if density function $f(x|a)$ does not satisfy the MLRP [*i.e.,* $f_a(x|a)/f(x|a)$ is not a monotonic function with respect to x for any a], then solving the problem will be harder.

Instead of analyzing the original CMP directly, we deal with one wherein (IR) is eliminated first, that is,

Relaxed CMP:

$$\hat{C}(a) \equiv \min_{w(x)\in W} \int w(x) f(x|a)dx \ \text{ subject to } \ \text{(IC)}.$$

By solving the relaxed CMP, we can obtain an efficient contract that motivates the agent to select target effort a.

Compared with the relaxed CMP, the original CMP has one more constraint, that is, constraint (IR). Notice that (IR) indicates whether the agent will accept a contract designed by the principal. If the agent accepts a solution to the relaxed CMP, this solution can also solve the original CMP. However, even if not, the CMP solution can solve the original CMP by adding a positive constant.

III. Analysis

Our first aim is to determine the optimal contract to solve the relaxed

CMP. The following proposition shows the suitability of a one-step bonus contract.

***Proposition* 1.** *The optimal contract to solve the relaxed cost minimization problem should be a one-step bonus contract.*

Proof. An expectation of any nondecreasing function is approximated by the sum of nondecreasing step functions. Thus, for an arbitrary sequence $\{x_i\}_{i\geq 0}$, where x_i is increasing in index i, the expectation of $w(x)$ given a is represented by the following:

$$E[w(x)\mid a] = w(x_0) + \sum_{i\geq 1} \Delta w_i \times [1 - F(x_i \mid a)], \tag{1}$$

where $\Delta w_i \equiv w(x_i) - w(x_{i-1})$, $i \geq 1$, is nonnegative because $w(x)$ must be nondecreasing in x. Differentiating Equation (1) regarding a provides the following:

$$\frac{\partial}{\partial a} E[w(x)\mid a] = \sum_{i\geq 1} \Delta w_i \times [-F_a(x_i \mid a)]. \tag{2}$$

Let $y_i = \Delta w_i \times [1 - F(x_i|a)] \geq 0$ for all $i \geq 0$. Thereafter, Equations (1) and (2) become the following:

$$E[w(x)\mid a] = y_0 + \sum_{i\geq 1} y_i,$$

and

$$\frac{\partial}{\partial a} E[w(x)\mid a] = \sum_{i\geq 1} y_i \times \frac{-F_a(x_i \mid a)}{1 - F(x_i \mid a)},$$

respectively. Thus, the relaxed CMP is rewritten as follows:

$$\min_{\{y_i\}_{i\geq 0}}, y_0 + \sum_{i\geq 1} y_i$$

$$\text{subject to} \quad \sum_{i\geq 1} y_i \times \frac{-F_a(x_i \mid a)}{1 - F(x_i \mid a)} = c'(a),$$

where $y_i \geq 0$ for all $i \geq 0$. If an index k exists such that $-F_a(x_i|a)/[1 - F(x_i|a)]$ has a positive maximum value, the solution is

$$y_k = \frac{1 - F(x_k \mid a)}{-F_a(x_k \mid a)} \times c'(a) > 0,$$

but $y_i = 0$ for all $i \neq k$, which implies $w(x_i) = 0$ for all $i < k$ and

$$w(x_i) = \frac{c'(a)}{-F_a(x_k \mid a)} \equiv b > 0$$

for all $i \geq k$. Q.E.D.

In the relaxed CMP, the principal aims to design the cheapest contract to motivate the agent to select target effort a. When the principal selects the threshold level to create such incentives, she will consider the effect compared with the cost. For instance, consider the situation that the principal gives the agent one more dollar to provide incentives when the return is not less than certain threshold x. At this time, the increment of wage cost, which the principal should pay in the expectation sense, is equal to $1 - F(x|a)$. Meanwhile, the incentive effect made from such a wage increase is $-F_a(x|a)$. Accordingly, the principal pays the cost of $1 - F(x|a)$ and obtains the incentive effect of $-F_a(x|a)$. Thus, the principal will select the threshold level where ratio $-F_a(x|a)/[1 - F(x|a)]$ is maximized and, at such a threshold level, provide one-shot incentives enough to induce target effort a.

The following lemma clarifies the threshold level characteristics where the ratio $-F_a(x|a)/[1 - F(x|a)]$ is maximized.

Lemma 2. *Let $x = x_c \in (\underline{x}, \bar{x})$ solve $-F_a(x|a)/[1 - F(x|a)] = f_a(x|a)/f(x|a)$. For any given a, $-F_a(x|a)/[1 - F(x|a)]$ increases from zero over interval $(\underline{x}, x_c)$, but decreases over interval $(x_c, \bar{x})$, which implies that $-F_a(x|a)/[1 - F(x|a)]$ has a positive maximum value at x_c.*

Proof. Let

$$x_m = \arg\max_x \frac{f_a(x \mid a)}{f(x \mid a)}.$$

For given a, the likelihood ratio $L(x) \equiv f_a(x|a)/f(x|a)$ has increasing interval $(\underline{x}, x_m)$ and decreasing interval $(x_m, \bar{x})$ by assumption. Differentiating $\phi(x) \equiv -F_a(x|a)/[1 - F(x|a)]$ yields the following:

$$\phi'(x) = \frac{f(x \mid a)}{1 - F(x \mid a)} \times [\phi(x) - L(x)],$$

which implies that both $\phi(x)$ is increasing for all x satisfying inequality $\phi(x) > L(x)$ and $\phi(x)$ is decreasing for all x satisfying inequality $\phi(x) < L(x)$. Moreover, $\phi(x)$ has critical points at the crossing points of the two functions.

Given that $\phi(\underline{x}) = 0$ because $F(\underline{x}|a) = 0$ and $F_a(\underline{x}|a) = 0$ and that $L(\underline{x}) = f_a(\underline{x}|a)/f(\underline{x}|a) < 0$ by the assumption on the likelihood ratio, we have $\phi(\underline{x}) > L(\underline{x})$. In addition, given that

$$\lim_{x \to \bar{x}} \frac{-F_a(x \mid a)}{1 - F(x \mid a)} = \frac{f_a(\bar{x} \mid a)}{f(\bar{x} \mid a)}$$

by the L'Hospital rule because $\lim_{x\to\bar{x}} F_a(x|a) = \lim_{x\to\bar{x}} [1 - F(x|a)] = 0$ and $f_a(\bar{x}|a)/f(\bar{x}|a) < 0$ by the assumption, we have $\phi(\bar{x}) = L(\bar{x}) < 0$. Based on these facts, we will show that $\phi(x)$ crosses $L(x)$ at $x = x_c$ from above only once, which implies $\phi(x)$ is increasing on interval $(\underline{x}, x_c)$ but decreasing on interval $(x_c, \bar{x})$. Thereafter, $\phi(x)$ has a positive maximum value at $x = x_c$.

Claim 1: $\phi(x)$ must cross $L(x)$ at least once.

Suppose $\phi(x)$ does not cross $L(x)$ at all, implying that, as $\phi(\underline{x}) > L(\underline{x})$, $\phi(x)$ is always greater than $L(x)$ [*i.e.*, $\phi(x) > L(x)$ for all $x \in (\underline{x}, \bar{x})$]. $\phi(x)$ is strictly increasing in x, which implies that $\phi(\bar{x}) > \phi(\underline{x}) = 0$. However, this inequality is contradictory to $\phi(\bar{x}) = L(\bar{x}) < 0$. Thus, $\phi(x)$ must cross $L(x)$ at least once.

Claim 2: $\phi(x)$ must cross $L(x)$ only once.

Suppose $\phi(x)$ crosses $L(x)$ at least twice. Denote x-coordinate of the i^{th} crossing point by x_i. Given that $\phi(\underline{x}) = 0$ and $L(\underline{x}) < 0$, we have $\phi(x) > L(x)$ for all $x \in (\underline{x}, x_1)$, but $\phi(x) < L(x)$ for all $x \in (x_1, x_2)$. Thus, $\phi(x)$ is increasing on interval $(\underline{x}, x_1)$, but $\phi(x)$ is decreasing on interval (x_1, x_2). In this case, $x_m < x_2$ should be accurate.[8] Given that $\phi(x)$ crosses $L(x)$ from below at

[8] This inequality can be obtained by making a contradiction. Suppose that $x_2 \leq x_m$. Given that $\phi(x)$ is decreasing on interval (x_1, x_2), we have $\phi(x_1) > \phi(x_2)$. However, given that $L(x)$ is increasing on interval $(\underline{x}, x_m)$ by assumption, $L(x_1) < L(x_2)$. Combining both inequalities with $\phi(x_2) = L(x_2)$ yields $L(x_1) < \phi(x_1)$, which

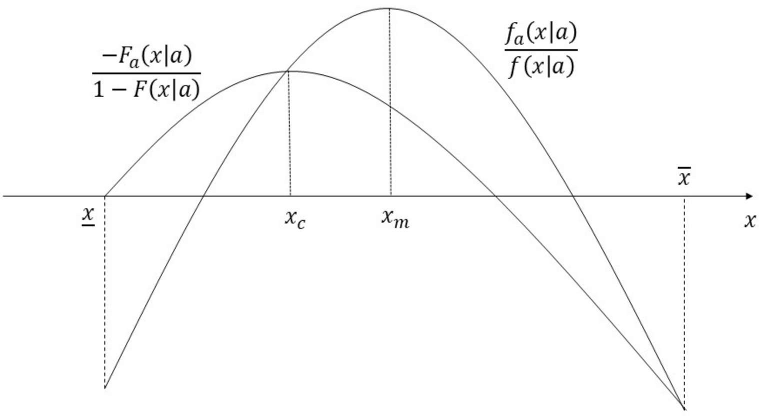

Figure 1
Graphs of $f_a(x|a)/f(x|a)$ and $-F_a(x|a)/[1-F(x|a)]$

x_2, and $L(x)$ is decreasing on interval $(x_m, \overline{x})$ by assumption, it must be true that $\phi(x) > L(x)$ for all $x \in (x_2, \overline{x})$, which means $\phi(x)$ increases over interval $(x_2, \overline{x})$. Considering $\phi(x_2) = L(x_2)$ implies that $\phi(\overline{x}) > L(\overline{x})$, which contradicts $\phi(\overline{x}) = L(\overline{x})$. Thus, $\phi(x)$ crosses $L(x)$ only once.

Denote the x-coordinate of the only crossing point by x_c.[9] $x_c \leq x_m$ should be accurate.[10] Given that $\phi(\underline{x}) = 0$ and that $\phi(x) > L(x)$ for all $x \in (\underline{x}, x_c)$, $\phi(x)$ increases starting from zero over interval $(\underline{x}, x_c)$, which implies $\phi(x_c) \equiv -F_a(x_c|a)/[1-F(x_c|a)] > 0$. Given $\phi(x) < L(x)$ for all $x \in (x_c, \overline{x})$, $\phi(x)$ decreases over interval $(x_c, \overline{x})$. Therefore, given that $\phi(x)$ increases from zero and then decreases to negative as x increases, $\phi(x_c) \equiv -F_a(x_c|a)/[1-F(x_c|a)]$ is the positive maximum value of $\phi(x)$. Q.E.D.

Lemma 2 shows mathematically that, under the assumption that the

contradicts the definition of x_1 (*i.e.*, $\phi(x_1) = L(x_1)$).

[9] Note that x_m or x_c may be a function of a. However, for simple notation, we suppress a.

[10] This inequality can be also obtained by making a contradiction. Suppose that $x_m < x_c$. Given that $\phi(x)$ is increasing on interval $(\underline{x}, x_c)$, $\phi(x_m) < \phi(x_c)$, and given that $L(x)$ has a maximum point at x_m, $L(x_m) > L(x_c)$. Combining both inequalities with $\phi(x_c) < L(x_c)$ gives $\phi(x_m) < L(x_m)$, which, together with $\phi(\underline{x}) = 0 > L(\underline{x})$, implies the existence of another $\hat{x} \in (\underline{x}, x_m)$ such that $\phi(\hat{x}) = L(\hat{x})$ by intermediate value theorem. This makes a contradiction.

likelihood ratio $f_a(x|a)/f(x|a)$ increases from negative and decreases to negative, the ratio $-F_a(x|a)/[1-F(x|a)]$ has a positive maximum value at the crossing point of two ratios $-F_a(x|a)/[1-F(x|a)]$ and $f_a(x|a)/f(x|a)$ (see Figure 1). This result is useful in understanding the following proposition.

Proposition 2. *Let $\hat{w}(x;\, a)$ the optimal contract to solve the relaxed cost minimization problem. Subsequently, for a given $a > 0$, $\hat{w}(x;\, a) = 0$ if $x < x_c$, and $\hat{w}(x;\, a) = b$ if $x \geq x_c$, where $x_c \in (\underline{x},\, \bar{x})$ satisfies $-F_a(x_c|a)/[1-F(x_c|a)] = f_a(x_c|a)/f(x_c|a)$, and $b = c'(a)/[-F_a(x_c|a)] > 0$. The compensation cost is $\hat{C}(a) = c'(a) \times [1-F(x_c|a)]/[-F_a(x_c|a)]$.*

Proof. In Proposition 1, given that the optimal contract for solving the relaxed CMP should be a one-step bonus contract, we consider the bonus contract, such as $w(x;\, t,\, b) = 0$ for $x < t$ and $w(x;\, t,\, b) = b$ for $x \geq t$. Thereafter, the principal aims to select a pair of $(t,\, b)$, thereby minimizing the expected wage cost $\int w(x;\, b,\, t) f(x|a)dx = b[1-F(t|a)]$ subject to $-bF_a(t|a) = c'(a)$. Inserting $b = c'(a)/[-F_a(t|a)] > 0$ into the objective function creates the following simple problem:

$$\min_{t \in X_0} c'(a) \times \frac{1-F(t\,|\,a)}{-F_a(t\,|\,a)},$$

where $X_0 \equiv \{x|F_a(x|a) < 0\}$. The optimal solution is t maximizing $-F_a(t|a)/[1-F(t|a)]$.

In Lemma 2, the function $-F_a(x|a)/[1-F(x|a)]$ has a positive maximum value at x_c, where $x = x_c$ solves equation $-F_a(x|a)/[1-F(x|a)] = f_a(x|a)/f(x|a)$. Thus, the optimal solution is $t = x_c$, and $b = c'(a)/[-F_a(x_c|a)]$. Equation

$$\hat{C}(a) = b[1-F(x_c\,|\,a)] = c'(a) \times \frac{1-F(x_c\,|\,a)}{-F_a(x_c\,|\,a)}$$

provides the compensation cost. Q.E.D.

Proposition 2 can be derived directly by combining the intuition of Proposition 1 and the result of Lemma 2. From Proposition 1, $-F_a(x|a)/[1-F(x|a)]$ means the ratio of the incentive effect to the wage cost when the principal increases the agent's wage at point x by one more dollar. Providing the agent with one-shot bonus at such a point that ratio

$-F_a(x|a)/[1-F(x|a)]$ is maximized is efficient for the principal. From Lemma 2, the ratio $-F_a(x|a)/[1-F(x|a)]$ is maximized at x_c which is the point where this ratio crosses the likelihood ratio. Therefore, the optimal bonus contract has the threshold x_c at which ratio $-F_a(x|a)/[1-F(x|a)]$ crosses likelihood ratio $f_a(x|a)/f(x|a)$, and the amount of bonus $b = c'(a)/[-F_a(x_c|a)]$, which is determined by (IC). Subsequently, the compensation cost is $\hat{C}(a) = b[1-F(x_c|a)] = c'(a) \times [1-F(x_c|a)]/[-F_a(x_c|a)]$.

The bonus contract $\hat{w}(x;\, a)$ to solve the relaxed CMP can be a solution to the original CMP. The following proposition shows the original CMP solution is a bonus contract obtained by adding a nonnegative constraint to the relaxed CMP solution.

Proposition 3. *Let* $\alpha \equiv \max\{c(a) + \bar{U} - E[\hat{w}(x;\, a)], 0\}$. $\hat{w}(x;\, a) + \alpha$ *is the optimal contract to solve the original cost minimization problem.*

Proof. Applying the Lagrange multiplier method to the original CMP gives the following:

$$\max_{\lambda,\, \mu \geq 0} \min_{w(x) \in W} \int w(x) f(x|a)dx - \lambda\left[\int w(x) f(x|a)dx - c(a) - \bar{U}\right] - \mu\left[\int w(x) f_a(x|a)dx - c'(a)\right],$$

where $W \equiv \{w(x)\,|\,w(x) \geq 0,\, w'(x) \geq 0\}$. Following Kuhn-Tucker's necessary conditions, either $\lambda = 0$ and $\int w(x)f(x|a)dx - c(a) - \bar{U} \geq 0$, or $\lambda > 0$ and $\int w(x)f(x|a)dx - c(a) - \bar{U} = 0$ should be satisfied at the optimum.

First, consider the case that $\lambda = 0$, which implies the original CMP is the same with the relaxed CMP. Hence, given that $\hat{w}(x;\, a)$ is the unique optimal contract to solve the relaxed CMP, $\hat{w}(x;\, a)$ is also the unique solution to the original CMP. Thus, if $E[\hat{w}(x;\, a)] \geq c(a) + \bar{U}$, then $\hat{w}(x;\, a)$ is the unique optimal contract to solve the original CMP.

Let us consider the case that $\lambda > 0$. This case implies that $E[\hat{w}(x;\, a)] < c(a) + \bar{U}$ is true, from which we have $\alpha = c(a) + \bar{U} - E[\hat{w}(x;\, a)] > 0$. By using $E[w(x)] = c(a) + \bar{U}$, the original CMP is reduced to the following: $\min_{w(x) \in W} c(a) + \bar{U}$ subject to $\int w(x) f_a(x|a)dx = c'(a)$. Any wage contract under which constraints (IC) and (IR) hold equal is a solution to this problem. Bonus contract $\hat{w}(x;\, a) + \alpha$ satisfies both constraints given that

$$\int[\hat{w}(x;\, a) + \alpha]f(x|a)dx - c(a) = \int \hat{w}(x;\, a)f(x|a)dx + \alpha - c(a) = \bar{U},$$

and

$$\int[\hat{w}(x;\ a) + \alpha]f_a(x|a)dx = \int\hat{w}(x;\ a)f_a(x|a)dx = c'(a),$$

where the first equality holds because $\int f_a(x|a)dx = 0$. Thus, $\hat{w}(x;\ a) + \alpha$ is one of the optimal contracts to solve the original CMP. Consequently, if $E[\hat{w}(x;\ a)] < c(a) + \bar{U}$, then $\hat{w}(x;\ a) + \alpha$ is an optimal contract to solve the original CMP. Q.E.D.

Proposition 3 shows that the bonus contract which is the unique solution to the relaxed CMP, plus a nonnegative constant, can solve the original CMP. Consider both the case where the bonus contract $\hat{w}(x;\ a)$ satisfies constraint (IR) and the case where it does not to understand what the Proposition 3 means precisely.

In the case that $E[\hat{w}(x;\ a)] - c(a) \geq \bar{U}$, we have $\alpha = 0$ given that $\bar{U} + c(a) - E[\hat{w}(x;\ a)] \leq 0$. Thus, Proposition 3 shows the bonus contract $\hat{w}(x;\ a)$ can also be a unique solution to the original CMP. This result is natural. The bonus contract $\hat{w}(x;\ a)$ is the cheapest contract only for incentive provision. Furthermore, if the agent accepts this contract [*i.e.*, it satisfies (IR)], then putting the cheap contract aside and designing other contracts do not make sense. Therefore, $\hat{w}(x;\ a)$ is also a unique solution to the original CMP.

Meanwhile, if $E[\hat{w}(x;\ a)] - c(a) < \bar{U}$, we have $\alpha = \bar{U} + c(a) - E[\hat{w}(x;\ a)] > 0$. Thereafter, Proposition 3 shows that $\hat{w}(x;\ a) + \alpha$ is one of the solutions of the original CMP. As explained in the proof of Proposition 3, an original CMP solution in this case is any wage contract that satisfies constraints (IC) and (IR) at the lowest cost. Given that $E[\hat{w}(x;\ a) + \alpha] - c(a) = \bar{U}$, and that $\int[\hat{w}(x;\ a) + \alpha]f_a(x|a)dx = \int\hat{w}(x;\ a)f_a(x|a)dx = c'(a)$, $\hat{w}(x;\ a) + \alpha$ satisfies both constraints at a minimum cost. Thus, $\hat{w}(x;\ a) + \alpha$ is one of the original CMP solutions. If the agent rejects the bonus contract $\hat{w}(x;\ a)$ which is the cheapest for incentive provision, the principal can design another bonus contract for the agent. Designing a new contract is possible by adding a positive constant α satisfying (IR) at a minimum to $\hat{w}(x;\ a)$. Thus, $\hat{w}(x;\ a) + \alpha$ becomes an original CMP solution.

Corollary 1. $C(a) = \max\{\hat{C}(a),\ c(a) + \bar{U}\}$, *where* $\hat{C}(a) = c'(a) \times [1 - F(x_c|a)]/[-F_a(x_c|a)]$.

The above corollary is derived from Proposition 3 directly. Recall that

$\hat{C}(a)$ is the value function of the relaxed CMP $\{$*i.e.,* $\hat{C}(a) \equiv E[\hat{w}(x;\ a)]\}$. If $E[\hat{w}(x;\ a)] \geq c(a) + \bar{U}$, then the unique solution to the original CMP is $\hat{w}(x;\ a)$, from which we have $C(a) = E[\hat{w}(x;\ a)] \equiv \hat{C}(a)$. Thus, if $\hat{C}(a) \geq c(a) + \bar{U}$, then $C(a) = \hat{C}(a)$. Meanwhile, if $E[\hat{w}(x;\ a)] < c(a) + \bar{U}$, then $\hat{w}(x;\ a) + \alpha$ is an original CMP solution, and its expectation $E[\hat{w}(x;\ a) + \alpha]$ is equal to $c(a) + \bar{U}$ as constraint (IR) is binding at the optimum in this case. Thus, when $\hat{C}(a) < c(a) + \bar{U}$, we have $C(a) = E[\hat{w}(x;\ a) + \alpha] = c(a) + \bar{U}$. Therefore, $C(a)$ is the maximum of $\hat{C}(a)$ and $c(a) + \bar{U}$.

IV. Justifying the First-Order Approach

Conditions justifying the first-order approach in the principal-agent problem where signal (*i.e.,* return in our model) is one-dimensional are found in Rogerson (1985), Jewitt (1988), and Jung and Kim (2015). However, existing conditions cannot be used in our model. The three sets of conditions in Rogerson (1985) and Jewitt (1988) include the MLRP, which is violated in our model because of the assumption regarding the likelihood ratio. Furthermore, the three sets of conditions in Jung and Kim (2015) are applicable only when the optimal contract is represented as a function of the likelihood ratio, which is violated because of the sabotaging ability of the agent in our model.[11] Thus, we must provide a new condition to validate the first-order approach in our model.

Proposition 4. *Let* $x_m = \mathrm{argmax}_x\ f_a(x|a)/f(x|a)$, *and let* $x_0 = \min\{x|f_a(x|a)/f(x|a) = 0\}$. *If* $F(x|a)$ *is decreasing and convex in* a *for all* $x \in (x_0,\ x_m]$, *the first-order approach is justified.*

Proof. In Proposition 3, the optimal contract for solving the cost minimization problem is $\hat{w}(x;\ a) + \alpha$, where $\hat{w}(x;\ a) = b \equiv c'(a)/[-F_a(x_c|a)]$ if $x \geq x_c$ and $\hat{w}(x;\ a) = 0$ if $x < x_c$ and $\alpha \geq 0$. Subsequently, given the optimal bonus contract $\hat{w}(x;\ a)$, and when the agent selects effort $\hat{a}$, his expected utility is the following:

$$EU_A(\hat{a}) \equiv E[\hat{w}(x;\ a)|\hat{a}] - c(\hat{a}),$$

where $E[\hat{w}(x;\ a)|\hat{a}] = b[1 - F(x_c|\hat{a})]$. Variable a refers to the target effort

[11] See the conditions of Propositions 1, 4, and 7 in Jung and Kim (2015).

level of the principal, and variable $\hat{a}$ means the effort choice level of the agent.

Given that $c(\hat{a})$ is increasing and convex in $\hat{a}$, if $E[\hat{w}(x;\ a)|\hat{a}]$ is increasing and concave in $\hat{a}$, then $EU_A(\hat{a})$ is concave in $\hat{a}$, which validates the first-order approach. Differentiating $E[\hat{w}(x;\ a)|\hat{a}]$ regarding $\hat{a}$ provides the following:

$$\frac{\partial}{\partial \hat{a}} E[\hat{w}(x; a) \mid \hat{a}] = -bF_a(x_c \mid \hat{a}), \tag{3}$$

differentiating once more gives

$$\frac{\partial^2}{\partial \hat{a}^2} E[\hat{w}(x; a) \mid \hat{a}] = -bF_{aa}(x_c \mid \hat{a}). \tag{4}$$

Note that $x_c \leq x_m$, as shown in the proof of Lemma 2. Given that $L(x_0) = f_a(x_0|a)/f(x_0|a) = 0$ by definition, that $L(x_c) = f_a(x_c|a)/f(x_c|a) = -F_a(x_c|a)/[1 - F(x_c|a)] > 0$ from Lemma 2, and that $L(x)$ is increasing on interval $(\underline{x},\ x_m)$, $x_0 < x_c$ is true. Thus, x_c always exists in interval $(x_0,\ x_m]$. If, for all $x \in (x_0,\ x_m)$, $F(x|a)$ is decreasing and convex in a, then the right hand side of Equation (3), $-\ bF_a(x_c|\hat{a})$, is positive and the right hand side of Equation (4), $-\ bF_{aa}(x_c|\hat{a})$, is negative, which means $E[\hat{w}(x;\ a)|\hat{a}]$ is increasing and concave in $\hat{a}$. Q.E.D.

The first-order approach can be justified by providing the conditions under which the expected utility of the agent under the optimal contract is concave in his effort choice. In our model, the agent is risk-neutral, and the optimal contract for solving the original CMP is $\hat{w}(x;\ a) + \alpha$, where $\alpha = \max\{c(a) + \bar{U} - E[\hat{w}(x;\ a)],\ 0\}$ in Proposition 3. Subsequently, when the agent is compensated with the optimal bonus contract $\hat{w}(x;\ a) + \alpha$ and chooses an arbitrary effort $\hat{a}$, the expected utility is equal to $EU_A(\hat{a}) = E[\hat{w}(x;\ a)|\hat{a}] + \alpha - c(\hat{a})$. Thus, given that $c(\hat{a})$ is increasing and convex in $\hat{a}$, the first-order approach is justified under the condition to guarantee that $E[\hat{w}(x;\ a)|\hat{a}]$ is increasing and concave in $\hat{a}$.

Note that $E[\hat{w}(x;\ a)|\hat{a}] = b[1 - F(x_c|\hat{a})]$, where x_c solves $-\ F_a(x|a)/[1 - F(x|a)] = f_a(x|a)/f(x|a)$ and $b \equiv c'(a)/[-\ F_a(x_c|a)] > 0$. Thereafter, the condition that $F(x_c|\hat{a})$ is decreasing and convex in $\hat{a}$ makes $E[\hat{w}(x;\ a)|\hat{a}]$ increasing and concave in $\hat{a}$. Nevertheless, x_c may not be calculated explicitly for a pre-specified distribution $F(x|a)$. To overcome this problem, we must find an interval on which x_c lies. As x_c is a maximum point of $-F_a(x|a)/[1-F(x|a)]$, x_c exists in interval $(x_0, x_m]$ (*i.e.*, $x_c \in (x_0, x_m]$) in Lemma 2. Thus, if $F(x|\hat{a})$ is decreasing and is convex in $\hat{a}$ for all $x \in (x_0, x_m]$, then $1 - F(x_c|\hat{a})$ is increasing and concave in $\hat{a}$, which implies that $E[\hat{w}(x; a)|\hat{a}]$ is increasing and concave in $\hat{a}$.

Certain examples can satisfy the condition that $F(x|a)$ is decreasing and convex in a for all $x \in (x_0, x_m]$ under the assumption that $f_a(x|a)/f(x|a)$ increases from negative and then decreases to negative as x increases.

***Example* 1.**

Normal distribution case. Consider the normal distribution: $\tilde{x} \sim N(\mu, \sigma(a))$, where $\sigma'(a) < 0$. Then, the likelihood ration is

$$\frac{f_a(x \mid a)}{f(x \mid a)} = \frac{[x-\mu]^2}{\sigma^3(a)}\sigma'(a) - \frac{\sigma'(a)}{\sigma(a)}.$$

Thus, we have $x_m = \mu$.

The cumulative distribution function is equal to the following:

$$F(x \mid a) = \Pr[\tilde{x} \le x \mid a] = \Pr\left[\tilde{z} \le \frac{x-\mu}{\sigma(a)}\right] = N\left(\frac{x-\mu}{\sigma(a)}\right),$$

where $\tilde{z} \equiv (\tilde{x}-\mu)/\sigma(a)$ and $N(z) = \int^z n(t)dt$ with $n(z) = 1/\sqrt{2\pi}\exp(-z^2/2)$. Note that $N'(z) = n(z)$ and $n'(z) = -z \times n(z)$. Thereafter, differentiating $F(x|a)$ regarding a provides the following:

$$F_a(x \mid a) = N'\left(\frac{x-\mu}{\sigma(a)}\right) \times \left[-\frac{x-\mu}{\sigma^2(a)}\right] \times \sigma'(a) = n\left(\frac{x-\mu}{\sigma(a)}\right) \times \left[-\frac{x-\mu}{\sigma^2(a)}\right] \times \sigma'(a) \le 0,$$

for all $x \le x_m = \mu$, where the inequality holds by $\sigma'(a) \le 0$. Differentiating $\ln[-F_a(x|a)]$ regarding a provides the following:

$$F_{aa}(x \mid a) = F_a(x \mid a) \times \left[\frac{n'\left(\frac{x-\mu}{\sigma(a)}\right)}{n\left(\frac{x-\mu}{\sigma(a)}\right)} \times (-\sigma'(a)) \times \frac{x-\mu}{\sigma^2(a)} - 2\frac{\sigma'(a)}{\sigma(a)} + \frac{\sigma''(a)}{\sigma'(a)}\right]$$

$$= F_a(x \mid a) \times \left[\frac{(x-\mu)^2}{\sigma^3(a)} \times \sigma'(a) - 2\frac{\sigma'(a)}{\sigma(a)} + \frac{\sigma''(a)}{\sigma'(a)} \right].$$

This equation shows that when $\sigma''(a)/\sigma'(a) \leq 2\ \sigma'(a)/\sigma(a)$, $F_{aa}(x|a) \geq 0$ for all $x \leq x_m = \mu$. Therefore, the first-order approach is valid under the condition that $\sigma''(a)/\sigma'(a) \leq 2\ \sigma'(a)/\sigma(a)$.

Example 2.

Convex mixture distribution case. Consider a convex mixture of any two symmetric distributions: $f(x|a) = \alpha(a)g(x) + [1 - \alpha(a)]h(x)$ with increasing and concave $\alpha(a) \in [0, 1]$, where $g(x)$ and $h(x)$ are symmetric probability density functions with similar mean μ. For our assumption, assume that $r(x) \equiv g(x)/h(x)$ is increasing on interval $(\underline{x}, \mu)$.

Given that $g(x)$ and $h(x)$ are symmetric around mean μ, $G(\mu) = H(\mu) = 1/2$ and $r(x) \equiv g(x)/h(x)$ is also symmetric about line $x = \mu$. Then, as $r(x)$ is increasing on interval $(\underline{x}, \mu)$ and decreasing on interval $(\mu, \bar{x})$, $r(x)$ has a maximum value at $x = \mu$.

Define $l(t) \equiv \alpha'(a)(t-1)/[1 + \alpha(a)(t-1)]$. We have

$$\frac{f_a(x \mid a)}{f(x \mid a)} = \frac{\alpha'(a)\left[\dfrac{g(x)}{h(x)} - 1\right]}{1 + \alpha(a)\left[\dfrac{g(x)}{h(x)} - 1\right]} = l(r(x)).$$

Given that $l(t)$ is an increasing function $\{i.e.,\ l'(t) = \alpha'(a)/[1 + \alpha(a)(t-1)]^2 > 0\}$ and that $r(x)$ is symmetric about line $x = \mu$, $f_a(x|a)/f(x|a) = l(r(x))$ is also symmetric. Thus, as $r(x)$ has a maximum value at $x = \mu$, $f_a(x|a)/f(x|a)$ also has a maximum value at $x = \mu$. This scenario indicates that $x_m = \mu$. Note that $f_a(\underline{x}|a)/f(\underline{x}|a) < 0$ and $f_a(x_m|a)/f(x_m|a) > 0$ from $E[f_a(x|a)/f(x|a)] = 0$, which implies that $r(\underline{x}) < 1$, $r(x_0) = 1$, and $r(\mu) > 1$.

Define

$$R(x) \equiv G(x) - H(x) = \int^x [g(t) - h(t)]dt = \int^x [r(t) - 1]h(t)dt.$$

From the definition, $R(\underline{x}) = G(\underline{x}) - H(\underline{x}) = 0$ and $R(\mu) = G(\mu) - H(\mu) = 0$. Given that $r(x)$ is increasing on interval $(\underline{x}, \mu)$, we have $r(x) \leq 1$ for all $x \in (\underline{x}, x_0)$, but $r(x) \geq 1$ for all $x \in (x_0, \mu)$. Given that $R'(x) = [r(x) - 1]h(x)$, $R(x)$ is decreasing on interval $(\underline{x}, x_0)$ but increasing on interval (x_0, μ). This scenario, together with $R(\underline{x}) = R(\mu) = 0$, implies that $R(x) \leq 0$ for all $x \leq \mu$.

The cumulative distribution function is equal to the following:

$$F(x|a) = H(x) + \alpha(a)[G(x) - H(x)] = H(x) + \alpha(a)R(x).$$

Subsequently, for all $x \leq \mu$,

$$F_a(x|a) = \alpha'(a)R(x) \leq 0,$$

where the inequality holds as $\alpha'(a) \geq 0$ and $R(x) \leq 0$ for all $x \leq \mu$, and

$$F_{aa}(x|a) = \alpha''(a)R(x) \geq 0,$$

where the inequality holds as $\alpha''(a) \leq 0$, and $R(x) \leq 0$ for all $x \leq \mu$. Hence, $F(x|a)$ is decreasing and convex in a for all $x \leq x_m = \mu$ under the condition that $\alpha(a)$ is increasing and concave, which justifies the first-order approach.

V. Conclusion

We consider the principal-agent problem where an agent assumes the role of a risk manager. We aim to find the optimal wage contract that will solve the cost minimization problem to induce any given effort. However, given that directly solving the original cost minimization problem is difficult because of the possibility that the individual rationality constraint may be nonbinding under the limited liability of the agent, we first analyze the relaxed cost minimization problem, wherein the individual rationality constraint is eliminated from the original cost minimization problem.

We affirmed that a solution to the relaxed cost minimization problem should be a one-step bonus contract, which indicates that a bonus contract is efficient in terms of incentive provision. Our result confirms that, if the bonus contract solving the relaxed cost minimization problem satisfies the individual rationality constraint of the agent, then it can also solve the original cost minimization problem. Otherwise, a new bonus contract, which is obtained by adding a positive constant to the bonus contract, can solve the original cost minimization problem.

If the individual rationality constraint is satisfied under the bonus contract for solving the relaxed cost minimization problem, it naturally becomes the unique solution to the original cost minimization problem, too. Otherwise, such a bonus contract cannot solve the original cost minimization problem because the agent will reject it. The reason is that the expected wage level of such a bonus contract is considerably low to satisfy the individual rationality constraint. Thus, by adding an appropriate constant which enables the individual rationality constraint to hold equal to such a bonus contract, the principal can design another bonus contract to solve the original cost minimization problem.

Finally, we provided the condition under which the first-order approach is valid. The first-order approach is justified under the condition to guarantee that the expected utility of the agent given the optimal bonus contract is concave in his effort. Thus, if the distribution is convex in effort for any outcome in the interval that the threshold level of the optimal bonus contract belongs to, then the agent's expected utility is concave in his effort, which validates the first-order approach. We presented certain examples that satisfy such a condition.

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References

Chava, S. and A. Purnanandam. "CEOs versus CFOs: Incentives and Corporate Policies." *Journal of Financial Economics* 97 (No. 2 2010): 263-278.

Coles, J. L., N. D. Daniel, and L. Naveen. "Managerial Incentives and Risk-taking." *Journal of financial Economics* 79 (No. 2 2006): 431-468.

Flaherty, J., G. Gourgey, and S. Natarajan. "Five Lessons Learned: Risk Management After the Crisis." *The European Financial Review* Retrieved on 30 April 2013, Available at http://www.europeanfinancialreview.com/?p=894.

Guay, W. R. "The Sensitivity of CEO Wealth to Equity Risk: An Analysis of the Magnitude and Determinants." *Journal of Financial Economics* 53 (No. 1 1999): 43-71.

Hirshleifer, D. and Y. Suh. "Risk, Managerial Effort, and Project Choice." *Journal of Financial Intermediation 2* (No. 3 1992): 308-345.

Innes, Robert D. "Limited Liability and Incentive Contracting with Ex-ante Action Choices." *Journal of Economic Theory* 52 (No. 1 1990): 45-67.

Jewitt, I. "Justifying the First-order Approach to Principal-agent Problems." *Econometrica* 56 (No. 5 1988): 1177-1190.

Jung, J. Y., and S. K. Kim. "Information Space Conditions for the First-order Approach in Agency Problems." *Journal of Economic Theory* 160 (2015): 243-279.

Kim, Son Ku. "Limited Liability and Bonus Contracts." *Journal of Economics & Management Strategy* 6 (No. 4 1997): 899-913.

______. "Optimal Reporting Frequency in Agencies." *Seoul Journal of Economics* 18 (No. 1 2005): 21-44.

Kini, O. and R. Williams. "Tournament Incentives, Firm Risk, and Corporate Policies." *Journal of Financial Economics*, 103 (No. 2 2012): 350-376.

Milgrom, Paul R. "Good News and Bad News: Representation Theorems and Applications." *The Bell Journal of Economics* 12 (No. 2 1981): 380-391.

Palomino, Frédéric and Andrea Prat. "Risk Taking and Optimal Contracts for Money Managers." *RAND Journal of Economics* 34 (No. 1 2003): 113-137.

Rogerson, W. P. "The First-order Approach to Principal-agent Problems." *Econometrica* 53 (No. 6 1985): 1357-1367.

Smith, C. W. and R. M. Stulz, "The Determinants of Firms' Hedging Policies." *Journal of Financial and Quantitative Analysis* 20 (No. 4 1985): 391-405.

Sung, J. "Linearity with Project Selection and Controllable Diffusion Rate in Continuous-time Principal-agent Problems." *The RAND Journal of Economics* 26 (No. 4 1995): 720-743.