Vertical Integration for Quality Signaling

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In the presence of consumers' incomplete information of firms' ability to produce quality components, we analyze firms' incentive to commit to a long-term relationship as a way to convince consumers about forming a high-type pair. In contrast to the result of no brand leverage obtained by Choi and Jeon (2007), our analysis demonstrates that a "brand-named" firm can restore its leverage by committing to a long-term relationship. To overcome the time inconsistency problem in a long-term contract, firms may utilize vertical integration with relation-specific investment. This signaling motivation for vertical integration is different from the explanations that currently exist.

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I. Introduction

Literature exploring the causes and effects of vertical integration has been extensive. Most have emphasized relationship-specific investments,

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incompleteness of contracts, and imperfect competition as possible causes for vertical integration.[1] These studies, however, disregard the incomplete information that consumers have about firms' productivities. In this context, our paper demonstrates that firms in a vertical relationship may have a reason for using a long-term contract or vertical integration with relation-specific investment to signal their capabilities to produce quality components.

We analyze vertical relationships in which production of a useful final good requires two complementary components of good quality. Each component is produced by one of the partner firms that belong to two different sectors. In each sector, two types of firms exist, namely the high- and the low-type firm: The former has a higher probability of producing a good-quality component than the latter. Consumers may infer which firm is accountable for a failed final product; they, however, have incomplete information about firm types. There are two periods, with each period being composed of a complete production and consumption cycle. Partner firms in period 1 may either continue or not their production partnership in period 2 depending on whether they sign a long-term or a short-term contract, respectively. Based on our analysis, the necessary and sufficient condition for a separating equilibrium, in which only pairs of high-type firms are productive, is weaker under a long-term contract than under a short-term one. This is so, if firms' probability of producing quality components is high enough. Therefore, firms in a vertical relationship may have an incentive to use a long-term contract as the leverage to convince consumers that they are forming a pair of high-type firms.

Our analysis also shows the *ex post* gain from voiding a long-term contract and engaging in re-matching with other firms in period 2. This, however, may threaten the credibility of a long-term contract as a commitment device for a long-term production relationship. As an alternative, we consider vertical integration with relation-specific investment that would eliminate the cause of engaging in re-matching with other

[1] Williamson (1971) advanced his seminal proposition that vertical integration is more likely when firms make relationship-specific investments, and Grossman and Hart (1986) developed a property-rights theory of integration in the context of an incomplete contract. From then on, a large volume of theoretical and empirical studies has explored the issue of vertical integration and long-term contracts. For a survey of such studies, see Gibbons (2005). With regard to literature on the strategic aspects of vertical integration under imperfect competition, see Rey and Tirole (2007), and Avenel (2008).

firms in period 2. Because the relation-specific investment is mainly used to convince consumers of a long-term production relationship, this signaling motivation for vertical integration that we suggest is different from the existing explanations for vertical integration.

Although this paper largely focuses on the case in which consumers have no problem in inferring the cause(s) of failure of their products, we show that the main result of the paper, the existence of an incentive for vertical integration for quality signaling, continues to hold even when consumers cannot infer the cause(s) of failure of their products. We discuss several cases of vertical integration as examples of vertical integration for quality signaling. Two of which are, the cases of Harim corporation, a major supplier of chicken meat products in Korea, and that of Swatch Group Korea, a Korean subsidiary of a multinational watch company, Swatch Group. The case of Korean whole life insurance market is also discussed to demonstrate the crucial role of foreign direction investment in creating a market for a certain product by enabling a separating equilibrium. The importance of a long-term contract or vertical integration for quality signaling may also be relevant in globalization; that is, firms have started to rely more on input supplies of foreign firms, of which consumers often have very limited information.

Our analysis is based on a simple production relationship model developed by Choi and Jeon (2007). They focused on the issue of "co-branding" that temporarily links firms in a new sector (*i.e.*, a sector of which consumers have incomplete information about firms' types) to the established firms with brand names in a mature sector. In particular, they analyzed whether such short-term co-branding may facilitate high-type firms in the new sector to signal their types so they can establish their own reputation (*i.e,.* whether an established firm can utilize its brand name as its leverage to convince consumers about its pairing with a high-type firm in a new sector under a short-term contract).[2] In

[2] Compared with the case of no mature sector (*i.e.*, no established firm to co-brand with), "co-branding" can relax the necessary and sufficient condition for a separating equilibrium *only when* "complementary between components (*i.e.*, a successful final product requiring good components from both sectors)" and "cross-sector inference problem (*i.e.*, consumers not being able to infer which firm is responsible for a failed final products)" exist. Tadelis (1999) developed a name trade model in an adverse selection framework with overlapping generations of firms. Choi and Jeon (2007) extended this name trade analysis of Tadelis (1999) into the analysis of a brand extension in multiple sectors with only one sector having brand names. Their analysis also differed from those of Tadelis (1999) and Cabral (2000) by relaxing the assumption of prices being

contrast to the result of no brand leverage obtained by Choi and Jeon (2007) in the absence of any cross-sector inference problem, our analysis demonstrates that a firm with a brand name can restore its brand leverage by committing to a long-term production relationship.

Our paper is organized as follows. Section 2 describes the setup of our model. Section 3 analyzes the case of a short-term contract, which essentially replicates the results of Choi and Jeon (2007). In Section 4, we first assume that firms can sign a long-term contract that forces partner firms in period 1 to continue their pairing in period 2, comparing the outcomes under such a longer-term contract with those under a short-term contract in Section 3. In a subsection of Section 4, we analyze the enforceability of a long-term contract and suggest vertical integration with relation-specific investment as an alternative way to commit a long-term production relationship. Section 5 shows that an incentive for signing a long-term contract or engaging in vertical integration exists even in the presence of cross-sector inference problem. It also discusses other robustness-related issues and provides examples of vertical integration for quality signaling. Section 6 presents the conclusion and discusses the possible extensions of our analysis.

II. Model

The basic setup of our model follows that of Choi and Jeon (2007). We consider a market for a final product that requires two complementary components, x and y. A continuum of firms producing component x has a mass normalized to 1. Another mass 1 of firms produces component y. We consider a two-period model in which each firm can produce at most one unit in each period. Consumers are homogeneous in their willingness to pay for the final product, and their mass is assumed as more than 1 in each period, implying that the sellers are on the short side of the market.[3]

equal to the consumers' willingness to pay, which in turn enables them to compare price signaling with brand signaling.

[3] Both assumptions, namely, consumers' being homogeneous and sellers being on the short side of the market are strong, as pointed out by a referee. These simplifying assumptions, which enable the second-period pricing to take a simple form, however, are not crucial in deriving the main result of the model; that is, the existence of an incentive for firms to engage in vertical integration for quality signaling. See Section 5 for a detailed discussion of robustness of vertical integration for quality signaling.

The quality of each component can be either good or bad. We assume that a final product is useful to a consumer if and only if both x and y components are of good quality. The value of a useful final product is normalized to 1. A final product, with any of its component being bad will render zero value to its consumer.

There are two types of firms, high (H) and low (L), in each sector, differ in their ability to produce a good quality component. More specifically, if a firm is H-type (L-type), it can produce a good component with probability $q_H(q_L)$ in period 1, with $1 > q_H > q_L \geq 0$. Per-period cost of producing a component is given by c_H and c_L for H-type and L-type firms, respectively, with $c_H > c_L > 0$, implying that L-type firms have a cost advantage over those of H-type. This generates an incentive to be paired with L-type firms for the purpose of saving costs.

The probability of each firm to make a good quality component in period 2 depends both on its type and on the quality of its component in period 1. For a θ-type firm with successful (failed) performance in period 1, $q_{\theta S}(q_{\theta F})$ denotes its probability of producing a good component in period 2, with $1 \geq q_{\theta S} > q_{\theta F} \geq 0$. If a θ-type firm does not produce any component in period 1, the probability of producing a high-quality component in period 2 is denoted by $q_{\theta N}$, with $q_{\theta N} < q_{\theta F}$. This inequality implies that a firm that does not produce in period 1 is penalized because it lacks experience useful for period 2 production. Let $\delta \in [0, 1]$ denote the discount factor for the second-period payoff, which is common for all firms.

In the following analysis, we will focus on the case where the following conditions are satisfied:

$$\begin{aligned} &(a) \quad (q_H)^2 - 2c_H > 0 > \max\left[(q_L)^2 - 2c_L, q_H q_L - c_H - c_L\right], \\ &(b) \quad q_{HF} q_{HF} - 2c_H > 0 > \\ &\qquad \max\left[q_{LS} q_{LS} - 2c_L, \max\left(q_{HS} q_{LS}, q_{LS} q_{HS}\right) - c_H - c_L\right]. \end{aligned} \tag{A1}$$

The above conditions imply that it is socially desirable for HH pair to produce while it is socially undesirable for any pair that involves an L-type firm to produce in any period.

Each firm recognizes its type and as well as the other firms'. Thus, we focus on incomplete information about firms' types only on the consumer side. With regard to consumers' information about firm types, we compare two cases. First, we consider the case where both sectors

are new; that is, the types of firms in neither sector are known to consumers at the beginning of period 1. This is a situation in which firms have yet to establish their reputation. Consumers only know the proportion of types. The proportion of H-type firms in sector i is given by $v_i \in (0, 1)$ with $i = x, y$. Second, we analyze the case in which only one of the two sectors, namely, sector x, is "mature," that is, a sector of which consumers already know the types of firms at the beginning of period 1. This may be interpreted as a case in which all H-type firms in x sector have good brand names, while those in sector have not established yet so consumers only know the proportion of types in sector y in period 1. In the two-new sector case, firms can only use price to signal their types, which is referred to as *price signaling*; on the contrary, in the one-mature-one-new sector case, firms can use their brand name to signal their product's quality, which is referred to as *brand signaling*.

With regard to cross-sector matching between firms, three different cases of cross-sector proportions of H-type firms, $v_x < v_y$, $v_x = v_y$, and $v_x > v_y$ generate different matching possibilities, which in turn may affect the outcome of the model. These different matching possibilities influence the bargaining power between partner firms, thus their payoffs.[4] However, the necessary and sufficient condition for a separating equilibrium in which firm types are revealed to consumers, the focus of our analysis, does not depend on different matching possibilities. Thus, we focus on the case with $v_x < v_y$, reflecting our presumption that brand names are likely to be a scarce resource. We also suppose that the matching process does not entail any search cost.

In contrast with Choi and Joen (2007) that focused on a short-term contract lasting for only one period, we assume that firms may use either a short-term contract or a long-term contract with their partners to subject them to be partners both in periods 1 and 2. At the end of each period, we assume that consumers can identify the true cause of failure: consumers can precisely observe the success or failure of each component after purchasing the product. Cho and Jeon (2007) referred this as a "*no cross-sector inference problem*" case because consumers have no problem in inferring the cause(s) of failure of their product.[5]

[4] See La and Park (2009), an earlier version of this paper, for the analysis of all three matching possibilities.

[5] They also analyze the case where consumers cannot infer the cause(s) of failure of their product, namely the "*cross-sector inference problem*" case. To check the robustness of vertical integration for quality signaling over different

In the absence of any cross-sector inference problem, they showed that the necessary and sufficient condition for a separating equilibrium with price signaling is identical to the corresponding condition with brand signaling. In the following analysis, we focus on the no cross-sector inference problem case, demonstrating that the equivalence result between price and brand signaling largely depends on their exclusive focus on the case of a short-term contract.

Each period is composed of five stages:

1. Firms search for their partners and are matched.
2. After matching, two firms decide whether to make a contract. If they decide not to, then they restart their costless search for partners (*i.e.*, go back to stage 1). Otherwise, they make either a short-term or a long-term contract.
3. The production takes place.
4. Each pair of firms is randomly matched to a consumer and makes a take-or-leave-it offer to the consumer. Each consumer decides whether to accept or to reject the offer. Before the offer, each pair of firms decides on the mode of signaling (either price or brand signaling) if brand signaling is an option.
5. After purchasing a product, consumers observe whether each component is of good or bad quality.

III. Benchmark Results under a Short-Term Contract

This section focuses on the case in which firms use a short-term contract with their partners, replicating the results of Choi and Jeon (2007). More specifically, we derive the necessary and sufficient condition for a separating equilibrium in which firm types are revealed to consumers through price (or brand if available) signaling.

***Period* 2:** To derive the condition for a separating equilibrium, we apply backward induction, describing first the equilibrium behavior in period 2 while assuming that only H-type firms are able to sell in period 1.[6]

assumptions on the consumers' informational constraint, Section 5 analyzes the cross-sector inference problem case, establishing that an incentive to sign a long-term contract or to engage in vertical integration for quality signaling continues to exist under this alternative assumption on the consumers' inference capability.

[6] Once firm types are revealed in a separating equilibrium, only pairs of H-type firms can make positive profits because the consumer's maximum willing-

Let p^2_{SF} be a period-2 price charged by the partners composed of an H-type in sector x with a success record (represented by the first subscript, S of p^2_{SF}) and an H-type in sector y with a failure record (represented by the second subscript F, of p^2_{SF}), with p^2_{SS}, p^2_{SF}, and p^2_{FF} being similarly defined. Then, the no cross-sector inference problem assumption and sellers being on the short side of the market imply the following values for these prices in period 2:

$$p^2_{SS} = (q_{HS})^2,\ p^2_{SF} = p^2_{FS} = q_{HS}q_{HF},\ \text{and}\ p^2_{FF} = (q_{HF})^2. \tag{1}$$

Although firms can freely pair in period 2 after the expiration of short-term contract at the end of period 1, no firm with a success record has an incentive to be in partner with a firm with a failure record in period 2. Such is "positive sorting" in which firms with the same record in period 1 match with each other in period 2. With the assumption of an equal division of revenue within an SS pair (a pair having success records in period 1) and within an FF pair (a pair having failure records in period 1) in period 2, such positive sorting is an equilibrium because $p^2_{SF}=p^2_{FS}=q_{HS}\,q_{HF}<(p^2_{SS}+p^2_{FF})/2=\{(q_{HS})^2+(q_{HF})^2\}/2$; this inequality implies that any deviation from positive sorting will yield a total payoff for a deviating pair that is strictly smaller than the sum of their individual payoffs under positive sorting, thereby forcing at least one of the deviating firms to have a strictly lower payoff.

***Period* 1:** Given the period-2 pricing behaviors and positive sorting described above, we can derive the condition for a separating equilibrium under two different signaling options, *price* and *brand signaling*. Prior to characterizing such conditions, we represent the present discounted value of a total joint payoff of a (short-term) pair composed of an i-type firm from sector x and a j-type firm from sector y with its period-1 price being p^1_{HH} by $V_{ij}(p^1_{HH})$:

$$\begin{aligned} V_{HH}(p^1_{HH}) &= p^1_{HH} - 2c_H + 2\delta[q_H(p^2_{SS}/2) + (1-q_H)(p^2_{FF}/2) - c_H], \\ V_{HL}(p^1_{HH}) = V_{LH}(p^1_{HH}) &= p^1_{HH} - c_H - c_L \\ &\quad +\delta[(q_H+q_L)(p^2_{SS}/2) + (2-q_H-q_L)(p^2_{FF}/2) - c_H - c_L], \end{aligned} \tag{2}$$

ness to pay for a final product is lower than the production cost for all other types of pairs that include at least one L-type firm, as assumed in (A1).

$$V_{LL}(p^1_{HH}) = p^1_{HH} - 2c_L + 2\delta[q_L(p^2_{SS}/2) + (1-q_L)(p^2_{FF}/2) - c_L].$$

In order to support a separating equilibrium in which only pairs of H-type firms are productive, the following individual rationality (IR) and incentive compatibility (IC) conditions need to be satisfied:

$$\begin{aligned}
&\text{for the case in which both sectors are new,}\\
&(\text{IR}^{\text{P}}) \quad V_{HH}(p^1_{HH}) \geq 0 \text{ and}\\
&(\text{IC}^{\text{P}}) \quad V_{HL}(p^1_{HH}) = V_{LH}(p^1_{HH}) \leq 0 \text{ and } V_{LL}(p^1_{HH}) \leq 0,\\
&\text{for the case in which sector x is mature and sector y is new,} \quad (3)\\
&(\text{IR}^{\text{B}}) \quad V_{HH}(p^1_{HH}) \geq 0 \text{ and}\\
&(\text{IC}^{\text{B}}) \quad V_{HH}(p^1_{HH}) \geq V_{HL}(p^1_{HH}),
\end{aligned}$$

with $p^1_{HH}=(q_H)^2$ under a separating equilibrium through brand signaling.[7]

For such a separating equilibrium, (IR) and (IC) are obviously necessary.[8] Given these conditions, we can derive the following benchmark results for a separating equilibrium under a short-term contract, which replicates the results of Choi and Jeon (2007).

***Benchmark Results* 1.** Given the set up described in Section 2, firms are assumed to sign a short-term contract when they match and produce their components for a final product.

a) When both sectors are new, a separating equilibrium through

[7] Under a separating equilibrium with brand signaling in period 1, $p^1_{HH}=(q_H)^2$ must hold by definition: if an HH pair can signal its type by the fact that an H-type firm of sector x is in its pair (branding signaling), then such an HH pair should be able to and will charge the consumer's maximum willingness to pay for its product, $(q_H)^2$. $p^1_{HH}=(q_H)^2$ implies $V_{HH}(p^1_{HH})>0$ from (A1), inducing $V_{HH}(p^1_{HH})\geq 0$ a redundant requirement.

[8] $V_{HH}(p^1_{HH})\geq 0$ is necessary for market participation of an HH pair to be individually rational. $V_{HL}(p^1_{HH})=V_{LH}(p^1_{HH})\leq 0$ and $V_{LL}(p^1_{HH})\leq 0$ are necessary to eliminate any L-type-involving pair's incentive to masquerade an HH pair's pricing behavior in period 1 when both sectors are new and there are some H-type firms that cannot pair with another H-type firm with $v_x<v_y$. When sector x is mature, (IC) no longer concerns about an LL pair's masquerading possibility, but $V_{HH}(p^1_{HH})\geq V_{HL}(p^1_{HH})$ is still necessary to eliminate the incentive of an H-type firm with a brand name to pair with an L-type firm, masquerading an HH pair's behavior in period 1 with $v_x<v_y$.

price signaling exists *iff (if and only if)* $\delta \geq \delta_C^P \equiv (c_H - c_L)/[(q_H - q_L)(p_{SS}^2 - p_{FF}^2)/2 - (c_H - c_L)]$. According to the Cho-Kreps refinement, $p_{HH}^1 = p_{HH}^{1P} \equiv (1+\delta)(c_H + c_L) - \delta\{p_{SS}^2(q_H + q_L)/2 + p_{FF}^2[1 - (q_H + q_L)/2]\}$.[9]

b) When sector x is mature and sector y is new, a separating equilibrium through brand signaling exists *iff* $\delta \geq \delta_C^B \equiv (c_H - c_L)/[(q_H - q_L)(p_{SS}^2 - p_{FF}^2)/2 - (c_H - c_L)]$, with $p_{HH}^1 = p_{HH}^{1B} \equiv (q_H)^2$.

Proof. See Appendix for the proof.

When both sectors are new, HH pairs signal their types under a separating equilibrium by setting a low enough price in period 1 so no incentive for HL, LH, or LL pairs are formed, masquerading as an HH pair (by setting the same low price). On the one hand, any pair with an L-type firm has a cost advantage over a HH pair because $c_H - c_L > 0$. On the other hand, an HH pair has an advantage over the others because an H-type firm has a higher probability for successful component production in period 1 than an L-type firm ($q_H - q_L > 0$). In addition, having a success record in period 1 enables a firm to set a higher price than a firm with a failure record under positive sorting in period 2 ($p_{SS}^2 - p_{FF}^2 > 0$), implying a higher expected joint revenue of period 2 for an HH pair. This higher expected joint revenue of period 2 may enable an HH pair to set a low enough price in period 1 to discourage other kinds of pairs to set the same low price. To support such price signaling as an equilibrium behavior, firms' valuation for future payoffs relative to current ones should be higher than the critical level with $\delta \geq \delta_C^P \equiv (c_H - c_L)/[(q_H - q_L)(p_{SS}^2 - p_{FF}^2)/2 - (c_H - c_L)]$. Note that it is easier to meet this condition, the smaller the value of $c_H - c_L$ and the bigger the values of $q_H - q_L$ and $p_{SS}^2 - p_{FF}^2$.

When sector x is mature and sector y is new, the necessary and sufficient condition for a separating equilibrium is the same as in the case of both sectors being new, with $\delta_C^P = \delta_C^B$. This may seem surprising because consumers know more about firm types when sector x is mature than when both sectors are new, possibly facilitating HH pairs' signaling their types.[10] The assumption of no cross-sector inference problem plays a key role in generating this equivalence result.[11] In the

[9] See Cho and Kreps (1987) for the definition and use of "Cho-Kreps refinement criterion."

[10] In fact, (3) partially confirms this intuition: Given (IR), (IC^P) implies (IC^B) but the reverse is not necessarily true.

[11] In the presence of a cross-sector inference problem, Choi and Jeon (2007) showed that the necessary and sufficient condition for a separating equilibrium

absence of any cross-sector inference problem, consumers can correctly infer each firm's performance in period 1 so that such performance affects its own payoff in period 2 but does not affect its partner's. This implies that each firm's expected individual payoff in period 2 is independent of its partner's type in period 1, which in turn makes each firm's net gain from pairing with an H-type rather than with an L-type firm in period 1 does not depend on the firm type, such that $V_{HH}(p^1_{HH}) - V_{HL}(p^1_{HH}) = V_{LH}(p^1_{HH}) - V_{LL}(p^1_{HH})$. As an H-type does not have any stronger incentive to pair with an H-type than an L-type firm has, the fact that consumers know a brand firm's type does not make its signaling for pairing with another H-type firm any easier than a no-brand H-type firm's signaling for such pairing.[12]

This equivalence result, however, does not imply that brand names do not have a role in the market. A firm with a brand name does benefit from it under a separating equilibrium through brand signaling. If $\delta \geq (c_H - c_L)/[(q_H - q_L)(p^2_{SS} - p^2_{FF})/2 - (c_H - c_L)]$, or equivalently $V_{HH}(p^1_{HH}) - V_{HL}(p^1_{HH}) \geq 0$, consumers will believe that an H-type firm in sector x has an incentive to pair with another H-type firm in sector y in period 1. In that case, an H-type firm with a brand name can set p^1_{HH} being equal to the consumers' reservation value for an HH pair's product in period 1, $(q_H)^2$. Because $p^{1B}_{HH} = (q_H)^2 > p^{1P}_{HH}$ with $(q_H)^2 > 2c_H > c_H + c_L$ and $p^2_{FF} > 2c_H$ from (A1), an H-type firm with a brand name sets a higher price than that without a brand name in period 1, realizing a higher expected

is weaker (*i.e.,* easier to be satisfied) under brand signaling than under price signaling. If consumers cannot figure out which component has contributed to the failure of a final product, an H-type firm has a stronger incentive to pair with an H-type firm than an L-type firm has for such pairing in period 1. This is because the increased likelihood of a success of its final product in period 1 from pairing with an H-type firm rather than with an L-type implies a higher expected individual payoff of period 2. Such an increase in the likelihood of a success in period 1 is higher for a H-type firm with $q_H(q_H - q_L) > q_L(q_H - q_L)$. Because consumers know that an H-type firm has a stronger incentive to pair with an H-type firm than an L-type firm has, an H-type firm with a brand name can more easily signal that it is paring with another H-type firm than an H-type with no brand name can.

[12] We can also provide a mathematical explanation for this result. Because $V_{HH}(p^1_{HH}) - V_{HL}(p^1_{HH}) = V_{LH}(p^1_{HH}) - V_{LL}(p^1_{HH})$, $V_{HH}(p^1_{HH}) - V_{HL}(p^1_{HH}) > 0$ implies that $V_{HH}(p^1_{HH}) - V_{LL}(p^1_{HH}) > 0$. As shown in the proof, the former inequality is the necessary and the sufficient condition for a separating equilibrium under brand signaling. The satisfaction of the former inequality implies both former and latter inequalities, the necessary and the sufficient condition for a separating equilibrium under price signaling, implying the equivalence result.

payoff under a separating equilibrium.

IV. A Long-Term Production Relationship for Quality Signaling

Benchmark Results 1 in the previous section have demonstrated that an H-type firm with a brand name cannot use its name value (*i.e.,* its type being already known to be H-type) as its leverage to convince its consumer that it is pairing with another H-type firm in the absence of any cross-sector inference problem. Although this is a surprising result, the analysis of the previous section exclusively focuses on a short-term production relationship between firms. This section explores the possibility of utilizing a long-term contract or vertical integration as a way to signal firm types. First, we assume that firms can sign a long-term contract that forces partner firms in period 1 to continue their partnership in period 2. The comparison of outcomes between those under a longer-term and those under a short-term contract is discussed. In the following subsection, we analyze the enforceability of a long-term contract and suggest vertical integration with relation-specific investment as an alternative way to commit to a long-term production relationship.

A. A Separating Equilibrium under a Long-Term Contract

In this subsection, firms are assumed to sign a long-term contract under which paired firms in period 1 should continue their partnership in period 2. Given this assumption, the necessary and sufficient condition is derived for a separating equilibrium in which only HH pairs are productive. To derive the condition for such a separating equilibrium, backward induction is applied, in which the equilibrium behavior in period 2 is described first.

***Period* 2:** Let p^2_{SS}, p^2_{SF}, p^2_{FS}, and p^2_{FF} be defined in the same way as in Section 3. Each represents a period-2 price set by a pair of firms with a different combination of records for their period-1 component production. Once again, the assumptions of no cross-sector inference problem and sellers being on the short side of the market imply the following values for these prices in period 2:

$$p^2_{SS} = (q_{HS})^2,\ p^2_{SF} = p^2_{FS} = q_{HS}q_{HF},\ \text{and}\ p^2_{FF} = (q_{HF})^2. \tag{4}$$

Because of a long term contract signed in period 1, re-matching among firms will not occur in period 2. As a result, "positive sorting" will not take place. With the assumption of (A1), all four possible types of pairs, namely, SS, SF, FS, and FF pairs, exist in terms of their success/failure records for period-1 component production, and sell their products in period 2.

***Period* 1:** Given the period-2 prices described above, we can derive the necessary and sufficient condition for a separating equilibrium under different signaling options. We represent the present discounted value of a total joint payoff of a long-term pair composed of an i-type firm from sector x and a j-type firm from sector y by $V_{ij}^{I}(p_{HH}^{1})$ with p_{HH}^{1} denoting its period-1 price:

$$\begin{aligned}
V_{HH}^{I}(p_{HH}^{1}) &= p_{HH}^{1} - 2c_H + \delta\{(q_H)^2 p_{SS}^{2} + 2q_H(1-q_H)p_{SF}^{2} \\
&\quad + (1-q_H)^2 p_{FF}^{2} - 2c_H\}, \\
V_{HL}^{I}(p_{HH}^{1}) &= V_{LH}^{I}(p_{HH}^{1}) = p_{HH}^{1} - c_H - c_L + \delta\{q_H q_L p_{SS}^{2} + [q_H(1-q_L) \\
&\quad + q_L(1-q_H)]p_{SF}^{2} + (1-q_H)(1-q_L)p_{FF}^{2} - c_H - c_L\}, \text{ and} \\
V_{LL}^{I}(p_{HH}^{1}) &= p_{HH}^{1} - 2c_L + \delta\{(q_L)^2 p_{SS}^{2} + 2q_L(1-q_L)p_{SF}^{2} \\
&\quad + (1-q_L)^2 p_{FF}^{2} - 2c_L\}.
\end{aligned} \tag{5}$$

In order to support a separating equilibrium in which only HH pairs are productive under a long-term contract, the following individual rationality (IR^{I}) and incentive compatibility (IC^{I}) conditions need to be satisfied:

for the case of both sectors being new,

(IR^{IP}) $V_{HH}^{I}(p_{HH}^{1}) \geq 0$ and,

(IC^{IP}) $V_{HL}^{I}(p_{HH}^{1}) = V_{LH}^{I}(p_{HH}^{1}) \leq 0$ and $V_{LL}^{I}(p_{HH}^{1}) \leq 0$,

for the case of sector x being mature and sector y being new, (6)

(IR^{IB}) $V_{HH}^{I}(p_{HH}^{1}) \geq 0$ and,

(IC^{IB}) $V_{HH}^{I}(p_{HH}^{1}) \geq V_{HL}^{I}(p_{HH}^{1})$,

with $p_{HH}^{1} = (q_H)^2$ under a separating equilibrium through brand signaling.

One can easily check that (IR^{I}) and (IC^{I}) are identical to (IR) and (IC)

in Section 3, with $V_{ij}(p^1_{HH})$ being replaced by $V^I_{ij}(p^1_{HH})$. With these (IR^I) and (IC^I) conditions, the following Proposition 1 can be derived for a separating equilibrium under a long-term contract:

***Proposition* 1.** Given the set up described in Section 2, firms are assumed to sign a long-term contract when they match and produce their components for a final product.

a) When both sectors are new, a separating equilibrium through price signaling exists *iff* $\delta \geq \delta_C^{IP} \equiv (c_H - c_L)/\{(q_H - q_L)[(p^2_{SS} - 2p^2_{SF} + p^2_{FF})(q_H + q_L)/2 + (p^2_{SF} - p^2_{FF})] - (c_H - c_L)\}$. By the Cho-Kreps refinement criterion, $p^1_{HH} = p^{1IP}_{HH} \equiv (1+\delta)(c_H + c_L) - \delta\{p^2_{SS} q_H q_L + p^2_{SF}[q_H(1-q_L) + q_L(1-q_H)] + p^2_{FF}(1-q_H)(1-q_L)\}$ when $V^1_{HL}(p^1_{HH}) > V^I_{LL}(p^1_{HH})$ and $p^1_{HH} = p^{1IP^*}_{HH} \equiv 2(1+\delta)c_L - \delta[p^2_{SS}(q_L)^2 + 2p^2_{SF} q_L(1-q_L) + p^2_{FF}(1-q_L)^2]$ when $V^I_{HL}(p^1_{HH}) \leq V^I_{LL}(p^1_{HH})$. $V^I_{HL}(p^1_{HH}) > V^I_{LL}(p^1_{HH})$ *iff* $\delta > (c_H - c_L)/\{(q_H - q_L)[(p^2_{SS} - 2p^2_{SF} + p^2_{FF})q_L + (p^2_{SF} - p^2_{FF})] - (c_H - c_L)\}$.

b) When sector x is mature and sector y is new, a separating equilibrium through brand signaling exists *iff* $\delta \geq \delta_C^{IB} \equiv (c_H - c_L)/\{(q_H - q_L)[(p^2_{SS} - 2p^2_{SF} + p^2_{FF})q_H + (p^2_{SF} - p^2_{FF})] - (c_H - c_L)\}$ with $p^1_{HH} = p^{1IB}_{HH} \equiv (q_H)^2$ so that $p^{1IB}_{HH} > p^{1IP}_{HH}$ and $p^{1IB}_{HH} > p^{1IP^*}_{HH}$.

Proof. See Appendix for the proof.

For Proposition 1, we can provide explanations similar to those for Benchmark Results 1 of Section 3. Under a long-term contract, pairing with an H-type firm rather than with an L-type firm continues to generate a higher expected joint revenue of period 2. This is because an H-type firm has a higher probability of producing a good component in period 1, and a partner firm's success record in period 1 will raise the consumer's willingness to pay for a final product in period 2. If δ is higher than a critical level (*i.e.,* firms' valuation of their period-2 payoff relative to their period-1 payoff is high enough), this benefit of a higher expected joint revenue of period 2 from pairing with an H-type can dominate the higher cost disadvantage of such pairing. This enables an HH pair to signal its type either by setting a low enough price in period 1 (price signaling) or by relying on the fact that one of the pairing firms is a known H-type firm (brand signaling). Proposition 1 specifies such a critical level of δ for price signaling in (a), denoting it by δ_C^{IP}, and for brand signaling in (b), denoting it by δ_C^{IB}.

Comparison of these critical levels of δ for price and brand signaling reveals that $\delta_C^{IP} > \delta_C^{IB}$ because $(q_H + q_L)/2 < q_H$, yielding the following corollary to Proposition 1:

***Corollary* 1.** If firms sign a long-term contract when they match and produce their components for a final product, the necessary and sufficient condition for a separating equilibrium is easier to be satisfied under brand signaling than under pricing signaling, with $\delta_C^{IB} < \delta_C^{IP}$.

This result is in contrast with Section 3's benchmark results under a short-term contract, in which brand names do not relax the necessary and sufficient condition for a separating equilibrium. If two firms sign a long-term contract in period 1, they cannot participate in the rematching process in period 2; hence, each firm's performance in period 1 affects not only its own payoff in period 2 but also its partner's. As a result, an H-type firm's net gain from pairing with an H-type firm rather than with an L-type firm in period 1 is greater than an L-type firm's corresponding net gain, with $V_{HH}^I(p_{HH}^1) - V_{HL}^I(p_{HH}^1) > V_{LH}^I(p_{HH}^1) - V_{LL}^I(p_{HH}^1)$; hence, a more productive firm gains more from pairing with a more productive firm under a long-term contract. Because a H-type firm has a stronger incentive to pair with another H-type firm than an L-type firm has for such pairing under a long-term contract, an H-type firm with a brand name can use its brand as leverage in convincing its consumer that it is forming an HH pair.

Signing a long-term contract may give an H-type firm with a brand name the leverage to convince its consumer about its partner being another H-type. However, whether firms would have a reason to sign an actual long-term contract remains unclear. This is because a long-term contract prohibits positive sorting in period 2, decreasing an HH pair's expected total joint payoff even lower than the one under a short-term contract: $V_{HH}^I(p_{HH}^1) < V_{HH}(p_{HH}^1)$. Therefore, firms will sign a long-term contract only when it is necessary for generating a separating equilibrium. Such situation can occur if the necessary and sufficient condition for a separating equilibrium is weaker under a long-term contract than under a short-term one.

As shown in the proofs of Benchmark Results 1 and Proposition 1, the necessary and sufficient condition for a separating equilibrium through price signaling is

$$\begin{aligned} &V_{HH}(p_{HH}^1) - V_{LL}(p_{HH}^1) = -2(1+\delta)(c_H - c_L) + \delta(q_H - q_L)(P_{SS}^2 - P_{FF}^2) \geq 0, \text{ and} \\ &V_{HH}^I(p_{HH}^1) - V_{LL}^I(p_{HH}^1) = -2(1+\delta)(c_H - c_L) \\ &\qquad + \delta(q_H - q_L)[(q_H + q_L)P_{SS}^2 - 2(q_H + q_L - 1)P_{SF}^2 \\ &\qquad + (q_H + q_L - 2)P_{FF}^2)] \geq 0, \end{aligned} \tag{7}$$

under a short-term and under a long-term contract, respectively. Similarly, the necessary and sufficient condition for a separating equilibrium through brand signaling is

$$\begin{aligned} V_{HH}(p^1_{HH}) - V_{HL}(p^1_{HH}) &= -(1+\delta)(c_H - c_L) \\ &\quad + \delta(q_H - q_L)(P^2_{SS} - P^2_{FF})/2 \geq 0, \text{ and} \\ V^I_{HH}(p^1_{HH}) - V^I_{HL}(p^1_{HH}) &= -2(1+\delta)(c_H - c_L) \\ &\quad + \delta(q_H - q_L)[q_H(P^2_{SS} - 2P^2_{SF} + P^2_{FF}) + P^2_{SF} - P^2_{FF}] \geq 0, \end{aligned} \tag{8}$$

under a short-term and under a long-term contract, respectively. Note the following relationships between these conditions:

$$\begin{aligned} &V^I_{HH}(p^1_{HH}) - V^I_{LL}(p^1_{HH}) - [V_{HH}(p^1_{HH}) - V_{LL}(p^1_{HH})] = \\ &\qquad \delta(q_H - q_L)(q_H + q_L - 1)(P^2_{SS} - 2P^2_{SF} + P^2_{FF}), \text{ and} \\ &V^I_{HH}(p^1_{HH}) - V^I_{HL}(p^1_{HH}) - [V_{HH}(p^1_{HH}) - V_{HL}(p^1_{HH})] = \\ &\qquad \delta(q_H - q_L)(q_H - 1/2)(P^2_{SS} - 2P^2_{SF} + P^2_{FF}). \end{aligned} \tag{9}$$

Because $q_H - q_L > 0$ and $p^2_{SS} - 2p^2_{SF} + p^2_{FF} > 0$ from (A1), (9) implies that

$$\begin{aligned} &V^I_{HH}(p^1_{HH}) - V^I_{LL}(p^1_{HH}) - [V_{HH}(p^1_{HH}) - V_{LL}(p^1_{HH})] > 0 \\ &\qquad \text{if and only if } q_H + q_L > 1, \text{ and} \\ &V^I_{HH}(p^1_{HH}) - V^I_{HL}(p^1_{HH}) - [V_{HH}(p^1_{HH}) - V_{HL}(p^1_{HH})] > 0 \\ &\qquad \text{if and only if } q_H > 1/2, \end{aligned} \tag{10}$$

which in turn implies the following results:

***Proposition* 2.** Assume that the set up is defined as in Section 2.

a) When both sectors are new, the necessary and sufficient condition for a separating equilibrium through price signaling is weaker under a long-term contract than under a short-term one *iff* $q_H + q_L > 1$.
b) When sector x is mature and sector y is new, the necessary and sufficient condition for a separating equilibrium through brand signaling is weaker under a long-term contract than under a short-term one *iff* $q_H > 1/2$.

Proposition 2 implies that extending the length of a contract can

facilitate H-type firms' signaling only when $q_H+q_L>1$ under price signaling and $q_H>1/2$ under brand signaling. If $q_H\leq 1/2$, for example, an H-type firm with a brand name does not have a reason to sign a long-term contract to signal that it is pairing with another H-type firm despite the leverage creating effect of a long-term contract demonstrated by Corollary 1. As shown in (8), an H-type firm's net gain from pairing with another H-type firm rather than with an L-type firm in period 1, increases in q_H-q_L under both types of contracts. This implies that generating a separating equilibrium is easier for an H-type firm with a brand name, the bigger the difference between q_H and q_L. After controlling this difference (*i.e.*, holding q_H-q_L at a fixed level), note that a decrease in q_H does not reduce an H-type firm's net gain from pairing with a more productive partner under a short-term contract, but the same decrease in q_H does reduce the net gain under a long-term contract: a less productive firm gets less from pairing with a more productive partner.[13] If q_H gets smaller than 1/2, then the necessary and sufficient condition for a separating equilibrium becomes more difficult to satisfy under a long-term contract than under a short-term one, thus providing no incentive for an H-type firm with a brand name to sign a costly long-term contract to generate a separating equilibrium.[14] We can provide a similar explanation for why H-type firms would not sign a long-term contract if $q_H+q_L\leq 1$ in the absence of brand names.

According to Proposition 2, a long-term contract is more likely for quality signaling in the presence of brand names than in their absence. This is because $q_H+q_L>1$ implies $q_H>1/2$, but the reverse is not true, with $q_H>q_L$. It is also worthwhile to note that $q_H+q_L>1$ will not hold when q_L gets too small, but $q_H>1/2$ is not affected by such a reduction in q_L. Thus, even when the success rate of an L-type firm is very low in period 1, eliminating the possibility of a long-term contract being signed under price signaling, an H-type firm with a brand name may still have an incentive to sign a long-term contract as long as its suc-

[13] Under a short-term contract, recall that each firm's net gain from pairing up with a more productive firm does not depend on its own type because each firm's individual performance in period 1 does not affect its period-1 partner's period-2 payoff as firms rematch through positive sorting in period 2. In contrast, each firm's net gain from such pairing does depend on its own type under a long-term contract, having each firm's net gain get higher, the higher the probability of its success in period 1.

[14] Recall that $V^I_{HH}(p^1_{HH})<V_{HH}(p^1_{HH})$ because firms cannot engage in profitable positive sorting in period 2 under a long-term contract.

cess rate in period 1 is higher than 1/2.

Using Proposition 2 together other results in our paper, we can consider various situations in which signing a long-term contract, possibly with a brand name, plays a crucial role in creating a market for a certain product by enabling a separating equilibrium. With regard to a foreign multinational firm's entry into a local market, for example, consider the case in which no local firm has a brand name with $\delta \in (\delta_C^{IB}, \delta_C^{IP})$ and $q_H + q_L > 1$.[15] Although signing a long-term contract relaxes the necessary and sufficient condition for a separating equilibrium through price signaling since $q_H + q_L > 1$ (Proposition 2a), local H-type firms without brand names cannot generate a separating equilibrium even if they sign a long-term contract because $\delta < \delta_C^{IP}$ (Proposition 1a). In the absence of local H-type firms with brand names, thus $\delta \in (\delta_C^{IB}, \delta_C^{IP})$ and $q_H + q_L > 1$ imply that no separating equilibrium can emerge through partnership between local firms. If a foreign multinational firm in sector x has a brand name, then it may consider entering the local market by making a partnership with a local firm. Note that even such a multinational firm with a brand name cannot generate a separating equilibrium if it signs a short-term contract with a local firm. This is because its brand name under a short-term contract does not create any leverage to convince consumers of its formation of an HH pair, as shown in the Benchmark Result 1.[16] With $\delta > \delta_C^{IB}$, however, a multinational firm with a brand name can generate a separating equilibrium by signing a long-term contract with a local firm. This implies creation of a market for a product of an HH pair, in which consumers are willing to pay a high price for the brand name.

B. Enforceability of a Long-Term Contract and Vertical Integration for Quality Signaling

While Propositions 1 and 2 demonstrate that signing a long-term contract instead of a short-term one can facilitate H-type firms' signaling

[15] As an example of this situation, we discuss the history of Korean whole life insurance market in Section 5.C. Prior to allowing foreign multinational firms' entry in life insurance market in 1988, the development in Korean market of sophisticated whole life insurance programs has been very limited.

[16] According to Proposition 2 (a), note that $q_H + q_L > 1$ implies $\delta_C^{IP} < \delta_C^P$. In addition, Benchmark Results 1 implies $\delta_C^B = \delta_C^P$, which further implies $\delta < \delta_C^{IP} < \delta_C^B$. Finally, note that $\delta < \delta_C^{IP} < \delta_C^B$ is compatible with $\delta_C^{IB} < \delta < \delta_C^{IP}$ because $q_H + q_L > 1$ implies $q_H > 1/2$.

of forming an HH pair to consumers, we have not questioned enforceability of a long-term contract by simply assuming it. Signing a long-term contract in period 1, however, does not necessarily guarantee that signing firms will continue to be partners in period 2. This is due to the positive gain that such firms may collectively obtain by voiding their contract to be long-term partners and engaging in positive sorting in period 2.

For partner firms that realize different outcomes in their component production (*i.e.,* one firm succeeds while the other fails) in period 1, their joint revenue in period 2 will be equal to $P^2_{SF}=P^2_{FS}=q_{HS}\,q_{HF}$ if they continue partnership.[17] If such a pair of firms decides to void its long-term contract and re-match with another pair of firms that had just the opposite outcomes in period 1, then the total joint revenue that these two pairs of firms can realize through positive sorting is $P^2_{SS}+P^2_{FF}=(q_{HS})^2+(q_{HF})^2$. Because $(q_{HS})^2+(q_{HF})^2>2q_{HS}\,q_{HF}$, voiding a long-term contract and re-matching through positive sorting will generate a net positive gain for such pairs. Thus, firms signing a long-term contract may suffer from time inconsistency, which would then nullify any potential signaling effect of a long-term contract.

An existence of a cost associated with voiding a long-term contract, however, may restore the credibility of a long-term production relationship. The maximum net benefit that any two pairs of firms can realize through positive sorting in period 2 is $(q_{HS}-q_{HF})^2$,[18] we can state the following result:

***Proposition* 3.** Assume the existence of a cost of voiding a long-term contract, denoted by F, that is common for any pair of firms. If $F>(q_{HS}-q_{HF})^2/2$, a long-term contract is enforceable.

[17] With regard to consumers' willingness to pay for a product in period 2, consumers are assumed to believe that only H-type firms are productive in period 1. Because we are still interested in obtaining the condition for a separating equilibrium, we continue such an assumption when we explicitly consider enforceability of a long-term contract.

[18] Prior to any re-matching in period 2, we can denote different types of paired firms according to paired firms' production records in period 1 by SS, SF, FS, FF, with the first (second) letter denoting the production record of a firm in sector x (y) and S (F) representing a success (failure) record. Then, there exist following six different combinations of re-matching possibilities among any two pairs of firms in period 2: (SS, FF), (SS, SF), (SS, FS), (FF, FS), (FF, SF), and (SF, FS). It is easy to check that only (SF, FS) may strictly benefit from re-matching, realizing a net benefit of $(q_{HS})^2+(q_{HF})^2-2q_{HS}\,q_{HF}=(q_{HS}-q_{HF})^2$, as shown in the preceding paragraph.

One way to create a cost of voiding a long-term contract is to require contracting firms to pay a third party, such as a local court, a fee that is greater than $(q_{HS} - q_{HF})^2/2$ if they void their long-term contract in period 2.[19] To convince consumers of the enforceability of a long-term contract, firms also need to publicize its existence. In practice, however, publicizing an inter-firm contract can be difficult as it may contain sensitive information, of which public revelation can be costly for contracting parties.

Vertical Integration for Quality Signaling

Given these potential difficulties in utilizing a long-term contract as a signaling device, H-type firms may consider vertical integration as an alternative way to signal that they are in a credible long-term production relationship. Vertical integration among firms often involves relation-specific investment that is irreversible or only partially reversible, potentially making future break-up a costly choice for integrating parties.[20] If the cost associated with breaking up a vertically integrated firm is higher than $(q_{HS} - q_{HF})^2/2$ in our model, then firms can credibly signal their long-term production relationship by vertical integration. Thus, our model suggests the possibility of forming a vertically integrated firm in which relation-specific investment is mainly for convincing the consumers about long-term relationship among its production units, which in turn would signal that its production units are H-type.

To illustrate the possibility of vertical integration for quality signaling, we construct a simple example of relation-specific investment that eliminates firms' ex post incentive for positive-sorting.

An Example of Relation-Specific Investment to Block Positive Sorting

The following relation-specific investment on a y-sector firm in period 1 will raise its probability of making a good component in period 2 only when its component is combined with a component of its period-1

[19] Any contractual arrangement involving only two firms that sign a long-term contract will also suffer from time inconsistency. This is because such firms can always come up with a re-arrangement in period 2 under which both of them can be strictly better off whenever they can gain from positive sorting in period 2. To eliminate the possibility of such an ex post re-arrangement among contracting firms, an enforceable long-term contract needs to include a thirty party who will implement the long-term contract.

[20] As discussed in the introduction, extensive literature on vertical integration and associated relation-specific investments exists.

partner. Denote such a raised probability of producing a good component in period 2 by $\hat{q}_{HS}(>q_{HS})$ after a success record in period 1 and by $\hat{q}_{HF}(>q_{HF})$ after a failure record. If the following condition in (11) is satisfied, then a pair of H-type firms that make such relation-specific investment in period 1 will not have any incentive to break up for positive sorting in period 2:

$$\begin{aligned} &q_{HF}\hat{q}_{HS} + q_{HS}q_{HF} > q_{HF}q_{HF} + q_{HS}q_{HS} \quad \text{and} \\ &q_{HS}\hat{q}_{HF} + q_{HF}q_{HS} > q_{HF}q_{HF} + q_{HS}q_{HS}. \end{aligned} \tag{11}$$

Recall that the incentive to break up for positive sorting in period 2 arises only when paired firms realize different outcomes in their component production in period 1 and such a pair meets with another pair of firms that had opposite outcomes in period 1. This continues to be true even when the y-sector firm's probability of making a good component in period 2 increases due to the relation-specific investment. The expressions on the left side of the inequalities in (11) represent the total joint revenue of two pairs of firms if they do not break up in period 2 and the expressions on the right side represent the total joint revenue of the same pairs of firms if they do break up for positive sorting. While the relation-specific investment would raise the presented discounted value of total joint revenue of a pair of H-type firms, the cost of such investment can be higher than the increase in the expected revenue, yielding a negative return on the investment. Even when the relation-specific investment yields a negative return, note that a pair of H-type firms may still make such investment to convince its consumers of their commitment to a long-term relationship if such commitment is necessary for them to signal their HH partnership.

To consider the relation-specific investment that is mainly for convincing consumers of a long-term production relationship, assume that the investment described above yields a negative return. Now, recall that $v_x < v_y$, thus a y-sector firm has no bargaining power with its x-sector partner in period 1; that is, a y-sector firm would sign any contract with an x-sector firm in period 1 as long as it guarantees a non-negative payoff.[21] An x-sector firm, based on such bargaining power, may

[21] Note that a y-sector firm would sign any contract with a x-sector firm in period 1 as long as it guarantees a non-negative (expected discounted) payoff does not contradict with Section 3's assumption of an equal division of revenue

offer the following long-term contract to a y-sector firm in period 1: the x-sector firm will provide the relation-specific investment for the y-sector firm in period 1, but the x-sector firm will only provide a payment to the y-sector firm just enough to compensate its marginal cost of production in both periods. It is hard to distinguish such a long-term contract from vertical integration because the contract practically deprives the y-sector firm of its right to exercise ownership over its asset, except for being compensated for its cost of producing the component good for the x-sector firm. Even when the y-sector firm is known to be an H-type firm in period 2, the long-term contract will prohibit the y-sector firm from using its brand power to realize any positive gain from it.[22] In addition, the x-sector firm makes relation-specific investment into the y-sector firm (or into its asset) to improve the latter's productivity — this usually takes place within a firm. Signing such a long-term contract, is equivalent to an x-sector firm's vertically integrating a y-sector firm with the relation-specific investment.[23]

This signaling motivation for vertical integration suggested by our analysis is quite different from existing explanations for vertical integration. As discussed in the introduction, previous studies explain the occurrence of vertical integration based on various reasons, such as a way to induce optimal relation-specific investment in the context of an incomplete contract, as a way to maneuver strategic actions under imperfect competition, and so on. In the given example of relation-specific investment, note that the investment yields a negative return, thus the primary reason for vertical integration with such investment is to signal that firms are forming an HH pair by credibly committing to a long-term production relationship.

within a SS pair and a FF pair after firms' types being revealed to consumers in period 2. In expectation of an equal division of revenue in period 2, which will generate some positive profit to a y-sector firm in period 2, a x-sector firm may offer a y-sector firm a contract that gives the y-sector firm a negative profit in period 1 so that its overall expected discounted profit is just equal to zero.

[22] Under a short-term contract, the relation-specific investment will strengthen the y-sector firm's bargaining power even further. This is because its x-sector partner in period 1 will have an incentive to keep their partnership due to the y-sector firm's increased probability of success in period 2, which implies a higher joint revenue in period 2 if they continue to be partners.

[23] Because the long-term contract is practically equivalent to vertical integration in its contents, we acknowledge that firms do not strictly prefer vertical integration over the long-term contract in the above example.

V. Robustness of Vertical Integration for Quality Signaling and Possible Examples

To test robustness of the result of vertical integration for quality signaling over alternative assumptions, the case of a cross-sector inference problem, as well as the discussion on other robustness-related issues, is presented in this section. The last subsection provides examples of vertical integration for quality signaling.

A. The Cross-Sector Inference Problem Case

To analyze the case of a cross-sector inference problem, we continue to assume the same set up as the one in Section 2, except assuming that consumers cannot infer the cause of failure of their products. First, we consider the case of short-term contract, indentifying the necessary and sufficient condition for a separating equilibrium. As shown by Choi and Jeon (2007), the prices in period 2 are:

$$p^2_{SS} = (q_{HS})^2,\ p^2_{SF} = p^2_{FS} = \frac{q_H(q_{HS})^2}{1+q_H} + \frac{q_{HS}q_{HF}}{1+q_H},\ \text{and}$$
$$p^2_{FF} = \frac{q_H(q_{HS})^2}{1+q_H} + \frac{(q_{HF})^2}{1+q_H}.^{24} \tag{12}$$

Given these prices, the following benchmark results are obtained for the case of a cross-sector inference problem by utilizing the same individual rationality and incentive compatibility conditions as those in Section 3:

***Benchmark Results* 2.** Given the set up described in Section 2, except that consumers cannot infer the cause(s) of failure of their products, firms are assumed to sign a short-term contract when they match and

[24] Note that we obtain p^2_{FF} in (12) by focusing on the most efficient equilibrium in which among the firms with a failure record, the successful producers of component x are matched with the successful producers of component y and similarly for the producers of failed components. p^2_{FS} in (12) represents consumers' maximum willingness to pay for a final product produced by a pair of a success-record firm and a failure-record firm, if consumers expect that the firm with a failure record is randomly chosen for matching with a success record. Moreover, $p^2_{FS} < (p^2_{SS} + p^2_{FF})/2$, implying that positive sorting will arise in period-2 matching.

produce their components for a final product.

a) When both sectors are new, a separating equilibrium through price signaling exists *iff* $\delta \geq \delta_{CI}^{P} \equiv 2(c_H - c_L)(1 + q_H)/\{(q_H + q_L)(q_H - q_L)[(q_{HS})^2 - (q_{HF})^2] - 2(1 + q_H)(c_H - c_L)\}$.

b) When sector x is mature and sector y is new, a separating equilibrium through brand signaling exists *iff* $\delta \geq \delta_{CI}^{B} \equiv (c_H - c_L)(1 + q_H)/\{q_H(q_H - q_L)[(q_{HS})^2 - (q_{HF})^2] - (1 + q_H)(c_H - c_L)\}$.

Proof. See Appendix for the proof.

To investigate whether signing a long-term contract or vertically integrating with relation-specific investment can facilitate H-type firms' signaling in the presence of a cross-sector inference problem, we now consider the long-term contract case, indentifying the corresponding necessary and sufficient condition for a separating equilibrium. Given that firms credibly commit to a long-term production relationship in period 1, only two types of prices exist in period 2, namely, the price of a pair with a success record in period 1 and the price of a pair with a failure record in period 1, as consumers cannot infer the cause(s) of failure of their products. Let p_{SS}^{2L} be a period-2 price charged by a pair of H-types with a success record and p_{FF}^{2L} be a period-2 price charged by a pair of H-types with a failure record; the superscript $2L$ denotes the period-2 price of a pair of firms in a long-term production relationship. Sellers being on the short side of the market implies the following values for these prices in period 2:

$$p_{SS}^{2L} = (q_{HS})^2 \text{ and } p_{FF}^{2L} = \frac{2q_H(q_{HS}q_{HF})}{1 + q_H} + \frac{(1 - q_H)(q_{HF})^2}{1 + q_H}. \tag{13}$$

Note that $p_{FF}^{2} > p_{FF}^{2L}$, which results from the commitment to a long-term production relationship that blocks formation of the most efficient equilibrium among the firms with failure records that had been feasible under a short-term contract.[25] As shown later, this difference in period-

[25] Footnote 24 describes formation of the most efficient equilibrium among firms with failure record. Similar to the case of no cross-sector inference problem analyzed in Section 4, firms that signed a long-term contract in the presence of a cross-sector inference problem may also have an incentive to void the long-term contract and form the most efficient equilibrium in the case of obtaining a failure record in period 1. Again, firms can overcome such a potential time-inconsistency problem either by credibly creating a fixed cost associated with voiding the long-term contract or by making relation-specific

2 pricing may enable H-type firms to signal their types by committing to a long-term production relationship. As in Section 4, the present discounted value of a total joint payoff of a long-term pair composed of an i-type firm from sector x and a j-type firm from sector y is represented by $V_{ij}^{I}(p_{HH}^{1})$ with p_{HH}^{1} denoting its period-1 price:

$$\begin{aligned} V_{HH}^{I}(p_{HH}^{1}) &= p_{HH}^{1} - 2c_H + \delta\{(q_H)^2 p_{SS}^{2L} + [1-(q_H)^2]p_{FF}^{2L} - 2c_H\}, \\ V_{HL}^{I}(p_{HH}^{1}) = V_{LH}^{I}(p_{HH}^{1}) &= p_{HH}^{1} - c_H - c_L + \delta[q_H q_L p_{SS}^{2L} \\ &\quad + (1-q_H q_L)p_{FF}^{2L} - c_H - c_L], \\ \text{and } V_{LL}^{I}(p_{HH}^{1}) &= p_{HH}^{1} - 2c_L + \delta\{(q_L)^2 p_{SS}^{2L} + [1-(q_L)^2]p_{FF}^{2L} - 2c_L\}. \end{aligned} \tag{14}$$

In consideration of these present discounted values of different types of long-term pairs, the following proposition for the case of a cross-sector inference problem can be obtained by utilizing the same individual rationality and incentive compatibility conditions as those in Section 4:

***Proposition* 4.** Given the set up described in Section 2, except that consumers cannot infer the cause(s) of failure of their product, firms are assumed to sign a long-term contract when they match and produce their components for a final product.

a) When both sectors are new, a separating equilibrium through price signaling exists *iff* $\delta \geq \delta_{CI}^{PL}$

$$\equiv \frac{2(c_H - c_L)(1+q_H)}{(q_H+q_L)(q_H-q_L)\{q_H[(q_{HS})^2 - 2q_{HS}q_{HF} + (q_{HF})^2] + [(q_{HS})^2 - (q_{HF})^2]\} - 2(1+q_H)(c_H - c_L)}.$$

b) When sector x is mature and sector y is new, a separating equilibrium through brand signaling exists *iff* $\delta \geq \delta_{CI}^{BL}$ with

$$\delta_{CI}^{BL} \equiv \frac{(c_H - c_L)(1+q_H)}{q_H(q_H-q_L)\{q_H[(q_{HS})^2 - 2q_{HS}q_{HF} + (q_{HF})^2] + [(q_{HS})^2 - (q_{HF})^2]\} - (1+q_H)(c_H - c_L)}.$$

Proof. See Appendix for the proof.

Comparing δ_{CI}^{P} with δ_{CI}^{PL}, and δ_{CI}^{B} with δ_{CI}^{BL}, the following proposition of

investment, similar to the ones described in Section 4.

vertical integration for quality signaling in the presence of a cross-sector inference problem can be obtained:

***Proposition* 5.** The set up is defined as in Section 2, except that consumers cannot infer the cause(s) of failure of their product.

a) When both sectors are new, the necessary and sufficient condition for a separating equilibrium through price signaling is weaker under a long-term contract than under a short-term one with $\delta_{CI}^{P} > \delta_{CI}^{PL}$.
b) When sector x is mature and sector y is new, the necessary and sufficient condition for a separating equilibrium through brand signaling is weaker under a long-term contract than under a short-term one with $\delta_{CI}^{B} > \delta_{CI}^{BL}$.

According to Proposition 5 (a), committing to a long-term production relationship enables firms without a brand name to convince consumers about their being H-types if $\delta \in [\delta_{CI}^{PL}, \delta_{CI}^{P})$. Similarly, Proposition 5 (b) shows that a firm with a brand name can restore its leverage to convince consumers that it is pairing with an H-type firm by committing to a long-term relationship if $\delta \in [\delta_{CI}^{BL}, \delta_{CI}^{B})$. Thus, Propositions 2 and 5 establish that the result of vertical integration for quality signaling is robust over different assumptions of consumers' ability to infer the cause(s) of failure of products. In fact, the necessary and sufficient condition for a separating equilibrium in the presence of a cross-sector inference problem is always weaker under a long-term contract than under a short-term one regardless of the firms' probability of producing quality components. This contrasts with Proposition 2 in the absence of any cross-sector inference problem.[26]

B. Other Robustness-Related Issues

The basic setup of the model described in Section 2 utilizes several simplifying assumptions, such as homogenous consumers, sellers' being on the short side of the market, and the existence of 2 periods only. Fully relaxing these assumptions, a means to check the robustness of vertical integration for quality signaling, is beyond the scope of this

[26] A referee suggested that the paper would be more convincing if we can show that the signaling motivation of vertical integration remains robust when we relax the assumption that consumers can observe individual firms' performance. We thank the referee for the comment, which led us to conduct this robustness check in Section 5.*A*.

paper. Nevertheless, we discuss issues associated with relaxing these assumptions in this subsection; herein, we provide our conjecture on how such relaxation might affect the result of this study.

The assumptions of homogenous consumers and sellers' being on the short-side of the market simplify our analysis by making the period-2 price of a final product depending only on the performance of firms in period 1. These assumptions also allow us to focus on the issue of signaling in the presence of incomplete information of firms' types, by eliminating strategic concerns among H-type pairs in setting the quantity (or, equivalently the price) of their products. One possible way to introduce heterogeneity among consumers is to consider a monopolistic competition model in which each pair of firms behaves as if they are a monopolist, thus we can continue to disregard the potential strategic concerns among H-type pairs in setting the prices of their final products. In such a setup, if H-type pairs can signal their types either through setting low enough prices or through their brand names, we can check if committing to a long-term production relationship could either facilitate or not the firms' signaling their types, in a manner similar to the one conducted in this paper. The cost and benefit associated with committing to a long-term relationship in such a model would be similar to the ones analyzed in this paper. Hence, we conjecture that the result of vertical integration for quality signaling would continue to hold in such a model with heterogeneous consumers.

Having only 2 periods is another simplifying assumption of our model. One may consider introducing additional periods into the model, potentially considering a model with infinite horizon. Given the setup of our model, we conjecture that introducing additional periods into the model would not affect the main result of our paper; that is, the existence of an incentive for vertical integration for quality signaling. For example, consider the case of no cross-sector inference problem with three periods to produce and consume final products. As the performance record in the second period would affect the likelihood of a successful performance in the third period, the third-period prices would be determined accordingly. With the third-period prices properly calculated, we may derive the present discounted value of a total joint payoff of the partner firms in the first period, with which we can characterize the necessary and sufficient condition for a separating equilibrium. If we continue to assume that pairing with a firm that has been inactive in the first period does not pay off, then the condition for a separating equilibrium would be relevant only for the first period. This is similar to the current model

with 2 periods. Committing to a long-term relationship will generate a cost similar to the case of the 2-period model, and such commitment can relax the necessary and sufficient condition under certain situations; a pair of H-type firms would have a lower joint expected payoff by committing to a long-term relationship but the reduction in the joint expected payoff can be bigger if such commitment is to pair with an L-type firm. Adding more periods, possibly infinitely, may further complicate the expressions for the total joint payoff of a pair of firms in the first period and the corresponding necessary and sufficient condition for a separating equilibrium. Nevertheless, there seems to be no reason for us to expect that having more periods will undermine the signaling incentive to sign a long-term contract, especially when firms can choose the duration of their contract.

The above discussion and the preceding subsection demonstrate that the result of vertical integration for signaling quality can be robust over different assumptions of the model, such as heterogeneous consumers, multiple periods, and consumers' having a cross-sector inference problem. This, of course, implies neither a signaling incentive for vertical integration will always exist nor generalizing the current model will not yield any interesting result. Possible ways to extend our current model are discussed in the concluding section.

C. Possible Examples of Vertical Integration for Quality Signaling

To provide possible examples of vertical integration for quality signaling, we discuss three cases of vertical integration. In the examples discussed below, firms engage in aggressive advertisement campaigns that emphasize their vertically-integrated production units as the companies' key characteristic, which together with some relation-specific investment would guarantee the quality of their final products.

a) The Case of Harim Corporation

Harim Corporation (Harim) is a major supplier of chicken meat-related products in Korea. Instead of outsourcing the production of feed for poultry and the production of chickens (raising chickens), Harim vertically integrated production units in upstream sectors, directly controlling the provision of chicks, feedstuff, raising procedures, and so on. This vertical integration, a major advertisement point of the company, may have strengthened consumers' confidence of its products, enabling it to have both higher prices and larger market shares than other firms

whose partners in the upstream sector are not vertically integrated.[27]

b) The Case of Swatch Group Korea

Swatch Group Korea Ltd. is a subsidiary company of the Swatch Group Ltd. (Swatch) in Switzerland. Swatch is the largest manufacturer and distributor of watches in the world. Initially, it entered the Korean market through an agency contract, which allowed such agency to sell its products anywhere in Korea. As a result, many Swatch products acquired a low-priced image in Korea. To change such image, Swatch has put Swatch Group Korea under its direct management, making Swatch products available in directly-managed stores. Swatch also built its largest A/S center in Korea. These actions of vertically integrating local service units in Korea have been instrumental in improving consumers' confidence on the quality of local service units, which in turn helped Swatch to restore its brand name (value) in the Korean market.[28]

c) The Case of the Life Insurance Market in Korea

With regard to a multinational firm's entry into a local market, we have considered the situation in which no local firm has a brand name with $\delta \in (\delta_C^{IB}, \delta_C^{IP})$ and $q_H + q_L > 1$ in Section 4.*A*.[29] In that situation, a long-term contract between a foreign firm with a brand name and a local firm is necessary for generating a separating equilibrium. Under such circumstance, prohibiting foreign direct investment may hinder market development in which consumers can buy products of HH pairs. For example, the Korean government had prohibited foreign direct investment into its life insurance market prior to 1988, limiting the provision of sophisticated whole life insurance programs in Korea.[30] Upon

[27] The company's website (http://www.harim.com/intro/about.hr), for example, explicitly emphasizes its vertically integrated production units as the main reason for consumers' trust on their products and its success in the chicken meat industry.

[28] The Swatch Group Korea's website (http://www.swatchgroup.co.kr/korean/aboutus_main.asp) emphasizes its mother company's direct investment into its A/S center in Korea and its direct control over distribution of its products as important characteristics of the company that would ascertain the quality of their products.

[29] Note that we can also consider a situation in which no local firm has a brand name with $\delta \in (\delta_{CI}^{BL}, \delta_{CI}^{PL})$ in the presence of a cross-sector inference problem.

[30] A sophisticated whole life insurance program requires well-trained employees to develop a tailored program to each customer, as well as a reliable com-

the liberalization of life insurance market in 1988, well-known foreign life insurance companies directly invested in the Korean market, providing sophisticated whole life insurance programs. The fact that a well-known life insurance company, such as Prudential Financial, Inc., has set a long-term relationship with its local sales unit by directly investing in Korea (vertical integration) could have strengthened consumers' belief of their local sales units' ability to tailor a whole life insurance program for the specific needs of each customer, thus enabling the provision of such service to the market.[31]

VI. Concluding Remarks

In the presence of consumers' incomplete information of the firms' capacity to produce good components for a useful final product, we analyze firms' commitment to a long-term production relationship as a possible way to convince consumers that they are engaged in partnership with high-type firms. In contrast to no brand leverage result of Choi and Jeon (2007) in the absence of any cross-sector inference problem, our analysis demonstrates that a firm with a brand name can restore its brand leverage by committing to a long-term production relationship.

pany that can handle a whole life insurance policy. As discussed by Grossman and Hart (1986), a whole life insurance company is more likely to vertically integrate its sales units (having the client list belong to the company rather than to its sales agency) possibly because of the typically high persistency of whole life insurance recipients. This in turn makes vertical integration an optimal ownership choice for a better combination of ante investment levels by the company and its sales units. In contrast to this incomplete contract approach to understand vertical integration, our analysis emphasizes the imperfect information of consumers about firm types as a possible driving force behind vertical integration in the Korean life insurance market.

[31] Prior to opening its life insurance market to foreign direct investments, sales of life insurance policies in Korea had been conducted mainly by temporary workers with low levels of training. In fact, Prudential Life Insurance Company of Korea, Ltd (Prudential Korea), the Korean subsidiary of Prudential Financial, Inc., was the first life insurance company in Korea that have only hired regular workers with at least a 4 year-college degree and some work experience for its sales units, calling them "life planners." In addition, Prudential Korea makes non-negligible investment into life planners' training (more than 6 months). As discussed in "an example of relation-specific investment to block positive sorting" above, we can interpret such training as investment to convince consumers of their long-term production relationship. The website of Prudential Korea (http://www.prudential.co.kr/company/lp_intro.do) advertises its stringent selection process of "life planners" and their training process.

Our analysis also reveals that the *ex post* gain from positive sorting may threaten the credibility of a long-term contract as an effective commitment device for a long-term relationship. As an alternative, we explore the possibility of vertical integration with relation-specific investment to restrain such positive sorting. This signaling motivation for vertical integration suggested by our analysis is different from the existing body of literature. Several cases of vertical integration as examples of vertical integration for signaling quality are also discussed.

Vertical integration for signaling quality may also be a reason for a company to choose foreign direct investment over foreign outsourcing. A firm may decide to move a part of its production process into a foreign country to save its production cost. With regard to partner's type in a foreign country, domestic consumers may have very limited information. Therefore, choosing foreign direct investment (a long-term production relationship) instead of outsourcing (a short-term production relationship) can facilitate the firm's signaling of its partnership with a high-type foreign firm.

There are several ways to extend the current model. For example, one can try to relax the simplifying assumptions. By relaxing the assumption of consumers being homogenous and sellers being on the short side of the market, we can allow the quantity demanded for a final product to depend on its price and other competing pairs' pricing as well as their signaling through vertical integration. Such a generalization may provide a new understanding of a possible linkage between vertical integration for quality signaling and other characteristics of the market, such as the nature of competition among firms.[32] The two-period model may also be extended into a multiple- or infinite-period model, characterizing the steady state characteristics of the signal game of firms producing complementary products. These extensions are nontrivial, thus, are possible directions for future research.

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[32] Once strategic interactions are introduced in the final product market under a more general setup, one may analyze the incentive for horizontal merger as well as the incentive for vertical integration. Although extensive literature exists on merger paradox, such as a study by Kabiraj and Lee (2003), an analysis of interaction between the incentive for vertical integration for quality signaling and the incentive for horizontal merger may lead to some new findings.

Appendix

***Proof for Benchmark Results* 1**

(a) First, we can show that a separating equilibrium through price signaling exists *iff* $V_{HH}(p^1_{HH}) \geq V_{HL}(p^1_{HH})(=V_{LH}(p^1_{HH}))$ and $V_{HH}(p^1_{HH}) \geq V_{LL}(p^1_{HH})$.

$V_{HH}(p^1_{HH}) \geq V_{HL}(p^1_{HH})$ and $V_{HH}(p^1_{HH}) \geq V_{LL}(p^1_{HH})$ must hold under a separating equilibrium from the necessary condition for the separating equilibrium described by (IR^P) and (IC^P). Thus, it remains to show that there exists p^1_{HH} which enables a separating equilibrium if $V_{HH}(p^1_{HH}) \geq V_{HL}(p^1_{HH})$ and $V_{HH}(p^1_{HH}) \geq V_{LL}(p^1_{HH})$. Let $\bar{p}^1_{HH}$ be defined by $\max\{V_{HL}(\bar{p}^1_{HH}), V_{LL}(\bar{p}^1_{HH})\} = 0$ and $\underline{p}^1_{HH}$ be defined by $V_{HH}(\underline{p}^1_{HH}) = 0$. Because $V_{HH}(p^1_{HH})$, $V_{HL}(p^1_{HH})$, $V_{LL}(p^1_{HH})$, and $V_{HH}(p^1_{HH})$ are increasing functions of p^1_{HH}, both $V_{HH}(p^1_{HH}) \geq V_{HL}(p^1_{HH})$ and $V_{HH}(p^1_{HH}) \geq V_{LL}(p^1_{HH})$ implies that $\bar{p}^1_{HH} \geq \underline{p}^1_{HH}$ holds. Then, a $p^1_{HH} \in [\underline{p}^1_{HH}, \bar{p}^1_{HH}]$ satisfies $V_{HH}(p^1_{HH}) \geq 0$, $V_{LL}(p^1_{HH}) \leq 0$ and $V_{HL}(p^1_{HH}) \leq 0$, enabling a separating equilibrium through price signaling in which only HH pairs are productive.

As $V_{HH}(p^1_{HH}) - V_{HL}(p^1_{HH}) = V_{HL}(p^1_{HH}) - V_{LL}(p^1_{HH})$, note that $V_{HH}(p^1_{HH}) \geq V_{HL}(p^1_{HH})$ implies $V_{HL}(p^1_{HH}) \geq V_{LL}(p^1_{HH})$, with $\bar{p}^1_{HH}$ being determined by $V_{HL}(\bar{p}^1_{HH}) = 0$. Among the continuum of separating equilibria with $p^1_{HH} \in [\underline{p}^1_{HH}, \bar{p}^1_{HH}]$, there is only one equilibrium that survives the Cho-Kreps refinement criterion, which is $p^{1P}_{HH} \equiv \bar{p}^1_{HH} = (1+\delta)(c_H + c_L) - \delta\{p^2_{SS}(q_H + q_L)/2 + p^2_{FF}[1 - (q_H + q_L)/2]\}$. To show this, suppose that a pair of H-type firms chooses p^1_{HH} lying in $[\underline{p}^1_{HH}, \bar{p}^1_{HH}]$ but different from $\bar{p}^1_{HH}$. Then such HH pair firms can deviate and choose a price of $p^1_{HH} + \varepsilon$, where ε is an infinitesimally small positive number. Since this new price satisfies the conditions of having a separating equilibrium, consumers would believe that this price has been chosen by a pair of H-type firms. Thus, H-type firms can profitably deviate from any $p^1_{HH} \in [\underline{p}^1_{HH}, \bar{p}^1_{HH})$ and the only reasonable equilibrium price in period 1 is $p^{1P}_{HH} \equiv \bar{p}^1_{HH}$.

Finally, note that $V_{HH}(p^1_{HH}) \geq V_{LL}(p^1_{HH})$ is equivalent to $V_{HH}(p^1_{HH}) \geq V_{HL}(p^1_{HH})$ because $V_{HH}(p^1_{HH}) - V_{LL}(p^1_{HH}) = 2[V_{HH}(p^1_{HH}) - V_{HL}(p^1_{HH})]$. This implies that $V_{HH}(p^1_{HH}) - V_{LL}(p^1_{HH})$ or equivalently, $\delta \geq (c_H - c_L)/[(q_H - q_L)(p^2_{SS} - p^2_{FF})/2 - (c_H - c_L)]$ is the necessary and sufficient condition for the existence of a separating equilibrium through price signaling, as claimed in Benchmark Result 1 (a).

(b) We can show that a separating equilibrium through brand signaling

exists *iff* $V_{HH}(p^1_{HH}) \geq V_{HL}(p^1_{HH})$. $V_{HH}(p^1_{HH}) \geq V_{HL}(p^1_{HH})$ must hold under a separating equilibrium from (IR^B) and (IC^B). Thus, it remains to show that $p^1_{HH} = p^{1B}_{HH} \equiv (q_H)^2$ is compatible with $V_{HH}(p^1_{HH}) \geq V_{HL}(p^1_{HH})$ and $V^I_{HH}(p^1_{HH}) \geq 0$ if $V_{HH}(p^1_{HH}) \geq V_{HL}(p^1_{HH})$, which is obviously true given (A1).

***Proof for Proposition* 1.**

(a) First, we can show that a separating equilibrium through price signaling exists *iff* $V^I_{HH}(p^1_{HH}) \geq V^I_{HL}(p^1_{HH}) (= V^I_{LH}(p^1_{HH}))$ and $V^I_{HH}(p^1_{HH}) \geq V^I_{LL}(p^1_{HH})$.

$V^I_{HH}(p^1_{HH}) \geq V^I_{HL}(p^1_{HH})$ and $V^I_{HH}(p^1_{HH}) \geq V^I_{LL}(p^1_{HH})$ must hold under a separating equilibrium from the necessary condition for the separating equilibrium described by (IR^{IP}) and (IC^{IP}). Thus, it remains to show that there exists p^1_{HH} which enables a separating equilibrium if $V^I_{HH}(p^1_{HH}) \geq V^I_{HL}(p^1_{HH})$ and $V^I_{HH}(p^1_{HH}) \geq V^I_{LL}(p^1_{HH})$. Let $\bar{p}^{1I}_{HH}$ be defined by $\max\{V^I_{HL}(\bar{p}^1_{HH}),\ V^I_{LL}(\bar{p}^1_{HH})\} = 0$ and $\underline{p}^{1I}_{HH}$ be defined by $V^I_{HH}(\underline{p}^{1I}_{HH}) = 0$. Because $V^I_{HL}(p^1_{HH})$, $V^I_{LL}(p^1_{HH})$ and $V^I_{HH}(p^1_{HH})$ are increasing functions of p^1_{HH}, any $p^1_{HH} \in [\underline{p}^{1I}_{HH},\ \bar{p}^{1I}_{HH}]$ satisfies $V^I_{HH}(p^1_{HH}) \geq 0$, $V^I_{LL}(p^1_{HH}) \leq 0$ and $V^I_{HL}(p^1_{HH}) \leq 0$, enabling a separating equilibrium through price signaling in which only HH pairs are productive. While we have a continuum of separating equilibria with $p^1_{HH} \in [\underline{p}^{1I}_{HH},\ \bar{p}^{1I}_{HH}]$, once again there is only one equilibrium that survives the Cho-Kreps refinement criterion, which is $p^1_{HH} = \bar{p}^{1I}_{HH}$ due to the same reason as the one given in the proof for Benchmark Result (a). If $V^I_{HL}(p^1_{HH}) > V^I_{LL}(p^1_{HH})$, $V^I_{HL}(\bar{p}^{1I}_{HH}) = 0$ defines $\bar{p}^{1I}_{HH}$ so that $p^1_{HH} = p^{1IP}_{HH} \equiv (1+\delta)(c_H + c_L) - \delta\{p^2_{SS} q_H q_L + p^2_{SF}[q_H(1-q_L) + q_L(1-q_H)] + p^2_{FF}(1-q_H)(1-q_L)\}$. If $V^I_{HL}(p^1_{HH}) \leq V^I_{LL}(p^1_{HH})$, then $V^I_{LL}(\bar{p}^{1I^*}_{HH}) = 0$ defines $\bar{p}^{1I}_{HH}$ so that $p^1_{HH} = p^{1IP^*}_{HH} \equiv 2(1+\delta)c_L - \delta[p^2_{SS}(q_L)^2 + 2p^2_{SF} q_L(1-q_L) + p^2_{FF}(1-q_L)^2]$. From $V^I_{HL}(p^1_{HH}) - V^I_{LL}(p^1_{HH}) = -(c_H - c_L) + \delta\{(q_H - q_L)[(p^2_{SS} - 2p^2_{SF} + p^2_{FF}) q_L + (p^2_{SF} - p^2_{FF})] - (c_H - c_L)\}$, $V^I_{HL}(p^1_{HH}) > V^I_{LL}(p^1_{HH})$ if and only if $\delta \geq (c_H - c_L) / \{(q_H - q_L)[(p^2_{SS} - 2p^2_{SF} + p^2_{FF}) q_L + (p^2_{SF} - p^2_{FF})] - (c_H - c_L)\}$.

Note that $V^I_{HH}(p^1_{HH}) \geq V^I_{LL}(p^1_{HH})$ implies $V^I_{HH}(p^1_{HH}) \geq V^I_{HL}(p^1_{HH})$ because $[V^I_{HH}(p^1_{HH}) - V^I_{HL}(p^1_{HH})] - [V^I_{HH}(p^1_{HH}) - V^I_{LL}(p^1_{HH})]/2 = \delta(q_H - q_L)^2(p^2_{SS} - 2p^2_{SF} + p^2_{FF}) \geq 0$. Thus, $V^I_{HH}(p^1_{HH}) \geq V^I_{LL}(p^1_{HH})$, or equivalently, $\delta \geq \delta^{IP}_C \equiv (c_H - c_L) / \{(q_H - q_L)[(p^2_{SS} - 2p^2_{SF} + p^2_{FF})(q_H + q_L)/2 + (p^2_{SF} - p^2_{FF})] - (c_H - c_L)\}$ is the necessary and sufficient condition for the existence of a separating equilibrium through price signaling, as claimed in Proposition 1 (a).

(b) We can show that a separating equilibrium through brand signaling exists *iff* $V^I_{HH}(p^1_{HH}) \geq V^I_{HL}(p^1_{HH})$. $V^I_{HH}(p^1_{HH}) \geq V^I_{HL}(p^1_{HH})$ must hold under a separating equilibrium where (IR^{IB}) and (IC^{IB}) are met. Thus, it remains to show that $p^1_{HH} = p^{1BI^*}_{HH} \equiv (q_H)^2$ is compatible with $V^I_{HH}(p^1_{HH})$

$\geq V^I_{HL}(p^1_{HH})$ and $V^I_{HH}(p^1_{HH}) \geq 0$ if $V^I_{HH}(p^1_{HH}) \geq V^I_{HL}(p^1_{HH})$, which is obviously true given (A1). Also note that $V^I_{HH}(p^1_{HH}) \geq V^I_{HL}(p^1_{HH})$ *iff* $\delta \geq \delta^{IB}_C \equiv (c_H - c_L)/\{(q_H - q_L)[(p^2_{SS} - 2p^2_{SF} + p^2_{FF})q_H + (p^2_{SF} - p^2_{FF})] - (c_H - c_L)\}$ from $V^I_{HH}(p^1_{HH}) - V^I_{HL}(p^1_{HH}) = -(c_H - c_L) + \delta\{(q_H - q_L)[(p^2_{SS} - 2p^2_{SF} + p^2_{FF})q_H + (p^2_{SF} - p^2_{FF})] - (c_H - c_L)\}$. Finally, $p^{1IB}_{HH} > p^{1IP}_{HH}$ and $p^{1IB}_{HH} > p^{1IB^*}_{HH}$ because $(q_H)^2 > 2c_H > (c_H + c_L) > p^{1IP}_{HH}$ and $(q_H)^2 > 2c_H > 2c_L > p^{1IP^*}_{HH}$ from (A1).

***Proof for Benchmark Results* 2.**

(a) As shown in Proof for Benchmark Results 1 (a), the separating equilibrium exists if and only if $V_{HH}(p^1_{HH}) \geq V_{HL}(p^1_{HH})$ and $V_{HH}(p^1_{HH}) \geq V_{LL}(p^1_{HH})$. Note that $V_{HH}(p^1_{HH}) - V_{HL}(p^1_{HH}) \geq V_{HL}(p^1_{HH}) - V_{LL}(p^1_{HH})$. Given this inequality, $V_{HL}(p^1_{HH}) \geq V_{LL}(p^1_{HH})$ implies that $V_{HH}(p^1_{HH}) \geq V_{HL}(p^1_{HH})$ and $V_{HH}(p^1_{HH}) \geq V_{LL}(p^1_{HH})$, satisfying the necessary and sufficient condition for the separating equilibrium. If $V_{HL}(p^1_{HH}) \geq V_{LL}(p^1_{HH})$, then the necessary and sufficient condition for the separating equilibrium is satisfied when $V_{HH}(p^1_{HH}) \geq V_{LL}(p^1_{HH})$ given the same inequality. $V_{HL}(p^1_{HH}) \geq V_{LL}(p^1_{HH})$ is equivalent to $\delta \geq (c_H - c_L)/\{q_L(q_H - q_L)(p^2_{SS} - p^2_{FF}) - (c_H - c_L)\}$ and $V_{HH}(p^1_{HH}) \geq V_{LL}(p^1_{HH})$ *iff* $\delta \geq 2(c_H - c_L)/\{(q_H + q_L)(q_H - q_L)(p^2_{SS} - p^2_{FF}) - 2(c_H - c_L)\}$. Finally, $(c_H - c_L)/\{q_L(q_H - q_L)(p^2_{SS} - p^2_{FF}) - (c_H - c_L)\} > 2(c_H - c_L)/\{(q_H + q_L)(q_H - q_L)(p^2_{SS} - p^2_{FF})$, which in turn implies that $\delta \geq 2(c_H - c_L)/\{(q_H + q_L)(q_H - q_L)(p^2_{SS} - p^2_{FF}) - 2(c_H - c_L)\}$ is the necessary and sufficient condition for the separating equilibrium under price signaling. Define $\delta^P_{CI} \equiv 2(c_H - c_L)(1 + q_H)/\{(q_H + q_L)(q_H - q_L)[(q_{HS})^2 - (q_{HF})^2] - 2(1 + q_H)(c_H - c_L)\}$ using $(p^2_{SS} - p^2_{FF}) = [(q_{HS})^2 - (q_{HF})^2]/(1 + q_H)$.

(b) As shown in Proof for Benchmark Results (b), the separating equilibrium exists if and only if $V_{HH}(p^1_{HH}) \geq V_{HL}(p^1_{HH}) \leftrightarrow \delta \geq (c_H - c_L)/\{q_H(q_H - q_L)(p^2_{SS} - p^2_{FF}) - (c_H - c_L)\}$. Thus, the necessary and sufficient condition for the separating equilibrium under brand signaling is $\delta \geq \delta^B_{CI} \equiv (c_H - c_L)(1 + q_H)/\{q_H(q_H - q_L)[(q_{HS})^2 - (q_{HF})^2] - (1 + q_H)(c_H - c_L)\}$.

***Proof for Proposition* 4.**

(a) We can apply the same statements as in the proof for Benchmark Results 2 (a) in proving Proposition 4 (a), except replacing $(p^2_{SS} - p^2_{FF})$ with $(p^{2L}_{SS} - p^{2L}_{FF})$, which leads to the following statement: $\delta \geq 2(c_H - c_L)/\{(q_H + q_L)(q_H - q_L)(p^{2L}_{SS} - p^{2L}_{FF}) - 2(c_H - c_L)\}$ is the necessary and sufficient condition for the separating equilibrium under price signaling. Using $(p^{2L}_{SS} - p^{2L}_{FF}) = (q_{HS} - q_{HF})[2q_H q_{HS} + (1 - q_H)(q_{HS} + q_{HF})]/(1 + q_H)$, we can obtain Proposition 4 (a).

(b) We can apply the same statements as in the proof for Benchmark Results 2 (b) in proving Proposition 4 (b), except replacing $(p^2_{SS} -$

p_{FF}^2) with ($p_{SS}^{2L} - p_{FF}^{2L}$). Thus, the necessary and sufficient condition for the separating equilibrium under brand signaling is $\delta \geq \delta_{CI}^{BL}$ with

$$\delta_{CI}^{BL} \equiv \frac{(c_H - c_L)(1+q_H)}{q_H(q_H - q_L)\{q_H[(q_{HS})^2 - 2q_{HS}q_{HF} + (q_{HF})^2] + [(q_{HS})^2 - (q_{HF})^2]\} - (1+q_H)(c_H - c_L)}.$$

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