Linear Production Function Based Modeling and Estimations of wages and profits in S. Korean economy: Consequential policies

Elias Sanidas

The introduction of productivity, either in the form of a function of labor productivity or a function of wages in a linear production function (LPF), opens numerous possibilities for the correct estimation of the main parameters of capital and labor prices, as well the capital and labor shares in the national economy. The LPF is also successfully combined with the Cobb–Douglas production function to confirm the estimations of these parameters. In addition, we provide evidence as to why the wage share in the economy has been declining. Several other relations are also generated, such as the constancy of the GDP to capital ratio. Moreover, some related policies are examined regarding wages and profits.

Keywords: economic growth, linear production function, Cobb–Douglas production function, wage, profit, capital share, labor share, constant trends, policies, productivity, panel data

JEL Classification: E10, E13, E19, E24, E60, E64, E69, O47, O49, O53

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I. Introduction

The initial motivation was to develop a methodology that explicitly includes some measurement of productivity into a production function (PF) with the ultimate aim to estimate econometrically a few parameters that seem more difficult or less rigorous in a standard Cobb–Douglas PF (CDPF). Thus, in this study, we develop some macro-economic relations based mainly on a linear PF (LPF), and combine the latter with the more often used CDPF. The use of the LPF opens new ways for establishing some long-term relationships between the main macro-economic variables and make some reliable econometric estimates.

Our theoretical and methodological content indicates that the CDPF is not after all the *bible* in macro-economic thinking. Moreover, one would dare to say that it should be used with caution as it does not adequately represent reality. For example, the usual assumption of constant returns to scale (CRS) in a macro CDPF are invalid. At the same time, we establish some "intrinsic relations" of an economy, which are related to the balanced growth of constant relations. In addition, we examine and estimate several equations that are related to the split between profits and wages.

The complementary purpose of this study is to pay equal attention to empirical estimates, as well as to theoretical considerations. Thus, we complement our modeling with consistent estimates of relevant regression coefficients. The consistency is primarily achieved through cross-checking the various alternatives available in the modeling of our intrinsic relations. For example, through the combination of the LPF and CDPF, we cross-check the labor and profit shares in the economy and the ratio of GDP (Y) to capital (K) (Y/K)."

In S. Korea and numerous other countries, the debate on the split of wages and profits is a major issue and usually generates several controversies. For example, in Joo *et al.*'s (2020) study, Chapter 2's title is "Is the Korean Economy in a Wage-led or Profit-led Growth Regime?" This debate is rather endless and without any definite conclusions, especially as it also touches on politics and political economics. For example, are the wages in S. Korea too low and profits too high? The present study is an attempt to clarify some relevant points by taking a different stand from the usual thinking. Particularly, we use LPF instead of CDPF. Thus, we believe that this study will give us many new analytical possibilities, which will become evident as we proceed.

However, this study is not intended to be exhaustive but rather an initial spark for further research. The main concern here is the empirical findings based on some theoretical issues, regardless of any particular school of economics (*e.g.,* neoclassical). We still refer to some standard issues as a comparison to our thinking.

According to our findings, overall, the markets in S. Korea have been working rather efficiently, such that wages and profits are not different from what one would have expected given the high growth rates of the S. Korean GDP in the last 65 years. However, some interesting points indicate that depending on the target of desired high growth, wages and profits should not change their relative position excessively from what they have been during this period, albeit with some significant qualifications. This is the policy content of the present paper.

In brief, the following questions will be explored and qualified with answers:

(i) Can we use the LPF?
(ii) Can we combine LPF with CDPF?
(iii) What is the capital price and the labor price?
(iv) What is the split between profit and wage shares in the economy?
(v) Particularly, what does it mean and how is it explained that the wage share is rather declining?
(vi) Do we have CRS or increasing returns to scale (IRS)?
(vii) Do we have a balanced growth whereby some of their economic variables or combinations of them are constant or nearly constant through time?

II. Models

A. Introduction

Technology and productivity (*A*), capital (*K*), and labor (*L*), either in terms of quality or quantity, are three fundamental factors (all other factors can stem from these three), which contribute to the economic growth of GDP (*Y*). All these factors may further increase productivity and technology. The importance of *A* is well-known in the literature (Acemoglu 2009; Skousen 1990; Schumpeter 1934; Sanidas 2004, 2005, 2006). *etc.* Acemoglu (2009) examined *A* mainly in the context of the neoclassical school in economics (See also Barro and Sala-I-Martin

2012).

Skousen (1990) examined in historical and non-theoretical terms the influence of technology on production. Skousen (1990, p.217) said, "…Both the critics and apologists for Bohm-Bawerk are, in my view, arguing about two aspects of the same phenomenon. In Taussig's as well as White's examples, there are two steps to new capitalistic methods: first, there is the building of the capital good, and second, the increased productivity that follows its implementation…" A more contemporary analysis of the productivity issue in economic growth can be found in Fraumeni (2019).

Particularly, productivity is considered in this study. Thus, we can approximate technology with productivity more directly (*e.g.*, with labor productivity ($LP = Y/L$)). Then, productivity can be used to examine PFs, which can establish theoretical and empirical relations. We explore this *LP* inclusion substantially in what follows. In addition, we use another proxy for productivity, namely, the wages in the economy. We avoid using TFP because it is mainly constructed with the CRS assumption; thus, it rather underestimates the true productivity in the economy.

As the title indicates, this study is not about supporting a particular type of PF, but rather it uses two types of PF to discuss the split between wages and profits, which, by definition, includes income from the rent of buildings. Overall, PF have several forms, which have been used in theoretical and empirical works (for a good historical account of theoretical PF, see Mishra (2007)).

Each one of these particular forms has advantages and disadvantages, and some of them are interrelated. Thus, the well-known CES function can be transformed into a linear or fixed proportion or Cobb–Douglas function, depending on the value of the CES parameters, mainly the elasticity of substitution between factors of production (Mishra 2007; Nicholson 1998). In this study, we use two forms of the CES, namely, the LPF ($LPF = aK + bL + \cdots$))[1] and the CDPF ($CDPF = Y = AK^{b}L^{1-b}$), which can also become linear if we take the logs of the variables involved.

In this study, we do not suggest that one form is superior over the other. On the contrary, the use of both shows the clarity of our intention

[1] Studies that used the LPF are Carboni and Russu (2013), Zhu (2000), and Walker (1971).

for no discrimination. Rather, we suggest that we can use both to our advantage in theoretical and empirical works, which is shown in more detail in the present study. Thus, when we use the CDPF, we indirectly assume that the coefficients of the linearized logarithmic CDPF are due to the constant elasticities of output with respect to capital and output with respect to labor, whereas the coefficients of the LPF express directly marginal products of capital and labor (Walker 1971).

Thus, we can directly estimate these marginal products via an LPF and hence estimate wages or profits, as well; this will be evidenced in the sections further below. In addition, the combination of the two types of linear functions (additive in K and L, and additive logarithmic in K and L as per CDPF) allows us to cross-check our findings (*e.g.*, whether the national production (GDP) has CRS). In other words, the cross-checking of the two PFs will bring more evidence to the validity of both PFs. Furthermore, the combination of LPF and CDPF will allow us to establish a CDPF that is more appropriate for IRS (see Equation (30)).

Another reason why we want to use LPF is that it is useful in understanding the Harrod–Domar (H–D) model, which we also use in our study. As Mishra (2007, p. 3) reported, "…The linear production function is important in view of the Harrod–Domar fixed coefficient model of an expanding economy and therefore every neoclassical production function, the Cobb-Douglas or its generalizations, must contain the linear production function $P = aK + bL$ to be consistent with it…" Effectively, we provide evidence of this issue here.

Moreover, we combine LPF and CDPF theoretically and empirically successfully, without discriminating one against the other. Furthermore, the possibility of exploring the additive LPF will allow us to include LP into the PF, something which is not possible or evident with the CDPF (because $LP = Y/L$ cancels out its factor L with the factor L of CDPF). The explicit inclusion of LP into the PF is important given that LP is the moving force of the GDP growth; without LP, growth can only be due to increases in population *ceteris paribus*. Moreover, notably, the Solow model can be derived from a linear model because it examines $LP = Y/L$ as a function of K/L, which is what we perform in this study in an explicit manner, starting from an LP (Equation 14), although our model is not exactly the Solow model.

In addition, the CDPF has been extensively used in the literature, probably because it was the first one to be utilized in the estimation of macro PFs. Here, we do not deny the usefulness of the CDPF, but we attempt to qualify its merits or disadvantages regarding the calculations of marginal products and thus capital and labor prices, as this is the main endeavor of the present study. My use of the CDPF in combination with the LPF provides evidence about the marginal products and hence the prices of K, L, and LP, as determined by the LPF.

Conversely, the LPF has been criticized (Nicholson 1998); particularly, given that the LPF has an infinite value of elasticity of substitution (ibid, p.304), utilizing it has become rather impossible because only one input (*e.g.*, *K*) will be used eventually given the appropriate relative prices of *K* and *L*. However, in practice, we can consider the case whereby input prices do not change considerably in the short run (*e.g.,* within a year); this reality tells us that there exists a continuous and slow substitution of labor for capital, which the LPF can easily handle within some boundaries. In addition, we can imagine a linear representation of the CDPF around some local values of *K*and *L*.

Furthermore, the complete or perfect substitution between *K* and *L* inherent in the LPF is only an approximation of the continuous substitution of *L* for *K*. On a macro-economic basis, this almost perfect substitution is considerably more plausible than on a micro basis because some branches and sectors of the economy may offset others. Another property of the CDPF, which is not present in the LPF, namely, the diminishing returns to *L* or *K*, again can be circumvented by the local linear approximation of the CDPF by the LPF in the year by year changes of the production variables.

Notably, given Uzawa's theorem regarding aggregate PFs, the CBPF in not necessary for balanced growth, and it is special and restrictive (Acemoglu, 2009, pp. 56–64). Here, balanced growth "...refers to an allocation where output grows at a constant rate and capital-output ratio, the interest rate, and factor shares remain constant..." (ibid, p. 57). This balanced growth[2] will be kept in mind in what follows, but it will not be restrictive to our thinking. In the same manner, the assumption of CRS so often encountered in the literature (such as

[2] Note that this concept of balanced (and the opposite of "uneven" growth) is not the same as the concept given in development economics, whereby uneven or unbalanced growth is the following development process: "...In many developing countries, economic growth has been fundamentally uneven. First one sector, then another, then a third have grown rapidly but not all together..." (Ray, 2010, p. 45).

Acemoglu's) will not be restrictive either.

B. LPF and marginal products

Let us then consider the following basic PF in a linear form:

$$Y = f(A, K, L) = aA + \beta K + cL, \tag{1}$$

where Y is the total output (*e.g.*, GDP), A is the productivity/technology in the economy, K is the stock of productive capital, and L is the stock of employment. The coefficients[3] of Equation (1) can be considered the marginal products of the three contributors (not factors, because A is not a factor of production, whereas K and L are), and are therefore the payments of their contribution; thus, c is wages, β is the capital price, and a is the productivity (*e.g.*, *LP*) price. We will see further below what these prices may be.

The total differentiation of Equation (1) yields

$$dY = adA + \beta dK + cdL, \tag{2}$$

which contains Equation (1), as we, for example, have $dY \sim \Delta Y = Y_t - Y_{t-1}$, and similarly for dK and dL.

Equation (2) amounts to

$$dY = MP_A dA + MP_K dK + MP_L dL, \tag{3}$$

where, for example, the marginal product of capital is

$$MP_K = f'_{K.} \tag{4}$$

Obviously, Equation (1) has CRS in terms of the two factors of production K and L, and A. However, depending on the actual form of A, for example, if we include A as an explicit variable, such as LP (Y/L) and not as a constant or as a time variable, then overall, Equation (1) may have IRS (which we will handle below).

From $dY = adA + \beta dK + cdL$ (2), we can obtain

[3] We use the Greek letter "β" instead of "b" for the coefficient of capital "K", because later (below), we will use "b" for the capital share in the economy (as part of the CDPF). However the remaining coefficients "a" and "c" are in Latin form.

$$\frac{dY}{dK} = a\left(\frac{dA}{dK}\right) + c\left(\frac{dL}{dK}\right) + \beta. \tag{5}$$

Having divided Equation (2) by dK as in Equation (5) and noticing that $dK = I$ (where I is investment), we have a long-term relationship between the slopes of our LPF Equation (1).

If we approximate A with LP, then

$$LP = \frac{Y}{L} \tag{6}$$

Hence, by totally differentiating Equation (6), we have

$$d\left(LP\right) = \frac{dY}{L} - \left(LP\right)\left(\frac{dL}{L}\right) \tag{7}$$

Then, by multiplying both sides by L and dividing by dK, we have

$$\frac{dY}{dK} = L\left(\frac{d\left(LP\right)}{dK}\right) + \left(LP\right)\left(\frac{dL}{dK}\right) \tag{8}$$

Comparing Equations (5) and (8) yields[4]

$$a = f\left(L\right) \tag{9}$$

$$c = f\left(LP\right). \tag{10}$$

To find β, we can use Equation (2) divided by dK and then compare it with Equation (5); and already knowing a and c, we obtain

$$\beta = f\left(\frac{Y}{K}\right). \tag{11}$$

To summarize these results, we can say that the coefficient of L or dL is related to LP, that is, the price of L is a function of LP, which includes wages; the coefficient of dLP is related to its price L; and the

[4] We prefer at this stage to express the coefficients as functions of the relevant variables; the justification will become apparent as we progress into the study.

coefficient of *dK* is related to the *Y/K* ratio. The approximation of *A* with *LP* makes estimating the payments *a*, *β* and *c* more difficult; hence, we use the approximations of some functions *f* in Equations (9)–(11). These approximations will be further clarified later, especially in the empirical section.

Thus, from Equation (1), we can calculate the marginal product of labor, that is,

$$MP_L = c = \frac{\partial Y}{\partial L} = f\left(LP\right) = f\left(\frac{Y}{L}\right) \tag{12}$$

Consequently, the marginal product of labor is equal to a function of the average product of labor (which is nothing surprising given our knowledge from micro-economics), that is, LP. We will see this more precisely when we show the relevant regressions.

Given that *LP* is used here, we should provide some clarifications about its meaning. On the basis of economic reasoning (Sanidas 2005), *LP* depends solely on two factors, namely, investment (*I*) which is technical innovations as embodied in capital (*K*) and organizational innovations or organosis (*O*), as per Sanidas (2005). Thus we may have the function *LP* = *f*(*I*,*O*), for example as in

$$LP = a_1 I^a O^b \Rightarrow gr\left(LP\right) = agr\left(I\right) + bgr\left(O\right),$$

where *gr* denotes growth. This notion is not the same as the TFP concept (empirically, the relationship between TFP and LP can be shown to be extremely strong as expected (for example, see Abel *et al.* (2011, Chapter 6), although the calculation of TFP is usually based on CRS. A further exploration of this issue is outside the scope of this paper).

C. Wages and profts

Another reason why we want to include *LP* explicitly (and for this purpose the LPF is the best choice) is the possibility of directly exploring the behavior of the time series of wages (*W*) and profits (Π) in the economy. Thus, consider the LPF by excluding the contributor LP. This new LPF, as shown in Equation (13), can be considered the cost function of the economy. However, as we will see further below, part of the coefficients *β* and *c* are used to generate the *LP*.

$$Y = f\left(K, L\right) = \beta K + cL \tag{13}$$

$$\frac{Y}{L}=\beta\frac{K}{L}+c\Rightarrow c\left(t\right)=W\left(t\right)=\frac{Y}{L}-\beta\frac{K}{L} \tag{14}$$

In the above equation, we transform the constant c into a variable depending on time and because we know from Equation (12) that the coefficient c represents wages. Thus, wages (W) are equal to a fraction only of $LP(Y/L)$, because the term $\beta(K/L)$ is subtracted from LP. Then, this subtracted term is the Π part of LP, where the coefficient β must be some sort of return (r) or (i) on the capital invested. Here, we can see something else of great importance, that is, the LP is made out of two components, the wages and the profits. In addition, notably, Equation (14) may remind us of the Solow model to some extent; the difference is the constant c in Equation (14), which is indicated by wages (W) here. Solow's model and its extensions (Berg 2001; Weil 2009) can be further adjusted to include some of the elements presented here. However, a detailed comparison with the Solow model is beyond the scope of this study.

However, we can find this return directly by deriving the complementary equation of Equation (14) based on Equation (13).

$$\frac{Y}{K}=\beta+c\frac{L}{K}\Rightarrow\beta\left(t\right)=i\left(t\right)=\frac{Y}{K}-c\frac{L}{K} \tag{15}$$

As for Equation (14), Equation (15) is made out of two components: profits and wages, given that in this case, the profit rate i is determined by subtracting the term $c(L/K)$ from Y/K. The two components should be similar in Equations (14) and (15). However, notably, an important difference exists between the two equations: in Equation (14), L is a stock variable measured in "items," that is, working people, whereas in Equation (15), K is only expressed in monetary terms and not in "items" (*e.g.*, machines). Hence, the interpretation should be different between the two equations.

Equations (14) and (15) are estimated econometrically.

We can also use Equation (14) in a different manner to see its relationship with the labor share out of the total income in the economy. Thus, from Equation (14), by multiplying both sides with L/Y (because $W(L/Y)$ = *labor share*), we obtain

$$W\left(\frac{L}{Y}\right)=\left(\frac{Y}{L}\right)\left(\frac{L}{Y}\right)-\beta\left(\frac{K}{L}\right)\left(\frac{L}{Y}\right)=1-\beta\left(\frac{K}{Y}\right)\Rightarrow\frac{L}{Y}=\frac{1}{W}\left(1-\beta\frac{K}{Y}\right). \tag{16}$$

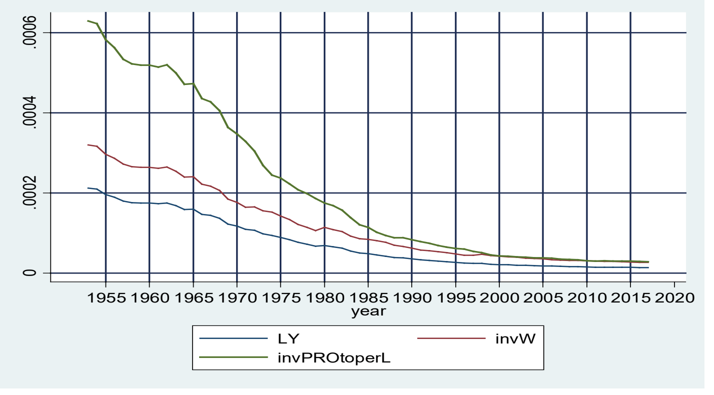
Equation (16) indicates that the ratio *L*/*Y*(labor over GDP) has the tendency to decline through time as the wages increase through time due to increases in productivity *Y*/*L*(and because the ratio *K*/*Y* in Equation (16) is constant through time; see further below for this constancy). More comments deserve our attention regarding Equation (16). First, the labor share in the economy tends to remain constant in an extremely long term (Cette *et al.* 2019; ILO/OECD 2015; Aum and Shin 2020).

According to our analysis, this result is due to the ratio *K*/*Y* remaining constant (see further below for evidence). However, the capital price (β) might have the tendency to decline (as we can witness in more mature economies, such as the USA, Canada, Germany, and France). Hence this decline of β counteracts the constancy of K/Y and the constancy of labor share. In S. Korea, K/Y has been increasing (see in Figure 2 its inverse, the Y/K ratio) and therefore, given the small changes in the capital price, the labor share has been declining but not continuously.

Second, as shown in Figure 1, the inverse of the wages per employed person, the inverse of the profits per employed person[5], and the ratio *L*/*Y* have all been declining (as per hyperbolic type) continuously since 1953, and they seem to converge especially after the end of the 1990s. This finding confirms our analysis regarding the long-term "balanced" growth of interdependence of variables. Seemingly, S. Korea is reaching a "steady state," whereby many variables will grow in parallel and with similar growth rates. Furthermore, wages and profits per employed person[6] are growing in tandem. Let us take the last period of the sample between 2000 and 2017 to elucidate the situation with an example. The *Y*/*K* ratio has declined from its average of 0.32 to 26%–27%. If we multiply this ratio by 13%, which is the expected capital price (as checked econometrically in the next section), then we obtain almost a 50%–50% split between wages and profits during this period (we already know that the capital productivity or *Y*/*K* ratio, in the same way as the LP *Y*/*L* is always split between wages and profits). Hence, we have convergence between wages and profits mentioned above. At the same

[5] On the basis of the labor share ("labsh") of the PWT, we can easily calculate W and profits per employed person.

[6] Of course, profits per company, and, especially per large companies such as chaebols, are considerably larger. A detailed analysis here is outside the scope of this study.



Note: The legend *invPROtoperL* (top line) indicates the inverse of total profit (see text above the graph for details) per employed person, *invW* (middle line) indicates the inverse of wages per employed person, and *LY* (lower line) represents the ratio L/Y.

Figure 1

Inverse of wages and profits per employee (labor-to-GDP ratio)

time, the wage share in the economy is also about 50%.

$labsh = 1 - 0.13 * 3.846 \approx 0.50$ by using Equation (16)

Furthermore, we can find the complement of wages in the economy, that is, the total profits (Π), which steadily grow through time, like the wages part

$$\Pi \quad share = 1 - laborshare = 1 - \left(1 - \beta \frac{K}{Y}\right) = \beta \frac{K}{Y} \tag{17}$$

This particular β is econometrically estimated. However, another way to arrive at the estimation of Π share is first, we can use some known series of investment return, such as the real internal rate of return provided by Penn World Table (PWT) data (irr). Then, the series of profits (Π) can be estimated as follows:

$$\Pi = (irr)(K) \tag{18}$$

from which the profit (Π) share can then be easily estimated as

$$\Pi \quad share = \frac{\Pi}{Y}$$

Obviously, we must cross-check our theoretical models with various relevant econometric results and against the actual data. We will do this cross-checking whenever possible.

D. Combination with CDPF

Now, suppose that we want to forcibly adopt a CDPF, as shown as follows:

$$Y = AK^{b}L^{1-b} \Rightarrow dY = MP_{A}dA + MP_{k}dK + MP_{L}dL, \tag{19}$$

where the coefficients of the factors of the total differential dY are the marginal products of these factors (A, which is technology and/or productivity), K is the stock of capital, and L is the stock of active labor force. These marginal products are equal to

$$MP_{A} = K^{b}L^{1-b}, \tag{20}$$

$$MP_{K} = A\left(b\right)\left(\frac{K}{L}\right)^{b-1}, \tag{21}$$

$$MP_{L} = A\left(1-b\right)\left(\frac{K}{L}\right)^{b}. \tag{22}$$

Note that the value of b in this CDPF has a different meaning from the previously used β. In the context of such function, b here denotes the share of capital K expenditure out of the total income (expressed as expenditure; Berg 2001).

The total differential in Equation (19) can also be derived for an LPF as we did earlier. Thus, on the basis of Equations (10) and (11), we can obtain the following:

$$MP_{K} = f\left(\frac{Y}{K}\right) = \alpha_{1}\left(\frac{Y}{K}\right), \tag{23}$$

$$MP_{L} = f\left(\frac{Y}{L}\right) = \alpha_{2}\left(\frac{Y}{L}\right) \tag{24}$$

Constants α_1 and α_1 are needed for the estimation procedure (see further below for their theoretical relations). We also need to determine what factor A can be. Generally, it represents technology and thus some sort of productivity. For empirical reasons, we should find a proxy for representing A. One such proxy is a function of wages, which depends on production, income, and productivity. Thus, we can approximate them (W) as follows:

$$W = f\left(production\right) = m'K^{b}L^{1-b}, \tag{25}$$

where m' is a correcting parameter. Thus, we can approximate A as follows:

$$A = f\left(W\right) = m''m'K^{b}L^{1-b} = mK^{b}L^{1-b}, \tag{26}$$

where m'' and m are correcting constants. Then, Equation (25) and m are econometrically estimated to show the validity of the proxy. Once we are satisfied with this approximation, then we can also estimate Equations (23) and (24) with the following two equations:

$$f\left(\frac{Y}{K}\right) = \alpha_1\left(\frac{Y}{K}\right) = mAbK^{b-1}L^{1-b} = mbK^{b}L^{1-b}K^{b-1}L^{1-b} = mbK^{2b-1}L^{2-2b}, \tag{27}$$

$$f\left(\frac{Y}{L}\right) = \alpha_2\left(\frac{Y}{L}\right) = mA(1-b)K^{b}L^{-b} = m\left(1-b\right)K^{b}L^{1-b}K^{b}L^{-b} = mbK^{2b}L^{1-2b}, \tag{28}$$

Thus, along with all these estimations, the constant b will also be estimated and cross-checked. In the next section, we will get a glimpse of the values of α_1 and α_2 as part of the estimation procedures. Apparently, after all the procedures, the empirical determination of b is paramount. The proper theoretical estimation of b (capital share) is intrinsically linked with the prices of capital and labor, as shown in Equations (27) and (28). All the three parameters can also be linked with one another through the following simple relationship:

$$\frac{Y}{L} = \left(\frac{Y}{K}\right)\left(\frac{K}{L}\right). \tag{29}$$

Equation (29) (identity) indicates that LP (Y/L) is equal to capital productivity (Y/K) multiplied by the capital-to-labor ratio (K/L). We will use this equation in the empirical section to further check on our various estimates.

Another conclusion is that the combination of LPF and CDPF can be fruitful for theoretical and empirical analyses. Although the CDPF has been used extensively in the relevant literature for many practical reasons or for "ideological" reasons (*e.g.*, adopted by the neoclassical school of economics), theoretically, we only show that these functions can be related, as indicated by Equations (27) and (28) and as the empirical section will confirm.

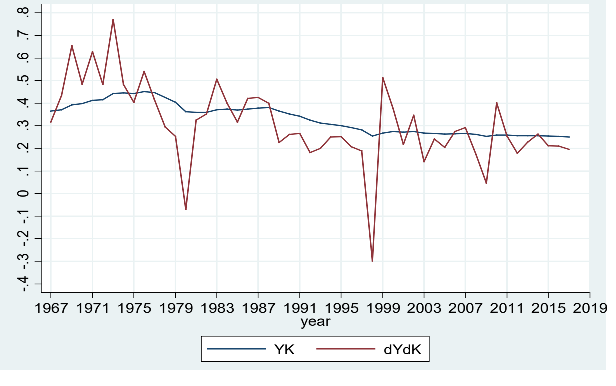
Given the form of the initial CDPF AK^bL^{1-b} and given the proxy for A, as shown in Equation (26), our form of CDPF suggested in this paper is

$$Y = mK^{2b}L^{2(1-b)}, \tag{30}$$

which has IRS in terms of K and L.

E. Long-term balanced growth (constancy of some variables)

Another aspect of some other properties of PFs that is potentially related to our theoretical and empirical results is as follows. In the literature (Acemoglu 2009), some ratios of production variables are constant or nearly constant through a long period of time, that is, the balanced growth axiom. One of these ratios is (Y/K). Figure 2 shows it for S. Korea.



Note: The period between 1955 and 1966 was excluded (for the sake of a clearer graph) from the graph as the $dYdK$ (=dY/dK) ratio exhibits extreme oscillations. The legend YK is the ratio Y/K (the smooth curve).

FIGURE 2

Y/K AND dY/dK RATIOS THROUGH TIME

Empirically, we will provide some evidence about these ratios. For now, we will develop some simple theoretical relations. First, the ratio *Y*/*K* is closely related to costs. Investors invest when they expect to regain at least what they invested. The three types of investment cost are: (a) the expected profit or return (in micro-economics, profit is considered as a cost), (b) the depreciation of capital, and (c) the expected labor costs during the initial (at least) years of production based on the new investment. The costs of intermediate goods can also be included as they represent a large part of production costs; however, given that we deal with macroeconomic variables that are based on value added, then we exclude these costs. We have some good idea of the above three costs.

In S. Korea, regarding the expected profit rate, the real internal rate of return (*irr*) can give us a good idea. As per PWT, it is about 9%–13% on average in the long period of 65 years (see below for its graph). The depreciation is about 3%–5% (see the PWT, variable "delta"). The labor costs can be as high as 15%. Conversely, the average of *Y*/*K* is about 32%. Given that the borrowing costs are already included in the calculation of GDP (*Y*), they are not part of the capital price. Hence, we can add 13% (profits) + 4% (depreciation) + 15% (labor), totaling 32%. Thus, in the long term, the tendency is to have about 32% covered costs from investments and labor (see below for more on this).

We can also provide some evidence of the *Y*/*K* constancy with some simple calculations as follows. We have in the case of this ratio's constancy:

$$YK = \frac{Y}{K} = \frac{Y}{L}\Big/\frac{K}{L} \Rightarrow \frac{dYK}{dt} = YK' = 0 \ \textit{for constancy.} \tag{31}$$

Thus, the first derivative of the ratio *YK* must equal zero for this to be constant. In other words, it reaches a minimum value that covers the costs of investments. This derivative, based on Equation (31), is

$$YK' = \left[\left(\frac{Y}{L}\right)'\left(\frac{K}{L}\right) - \left(\frac{K}{L}\right)'\left(\frac{Y}{L}\right)\right]\Big/\left[\left(\frac{K}{L}\right)^2\right] = 0 \Rightarrow \ldots E(K,Y) = \frac{\left[1+E(L,Y)\right]}{\left[1+E(L,K)\right]'} \tag{32}$$

Where $E(K, Y) = (dK / K)/(dY / Y)$ stands for elasticity of K in relation to Y. This relationship tells us that the constancy of the *Y*/*K* ratio takes place when the elasticity of K in relation to Y is constant and equal to one plus the elasticity of L in relation to Y divided by one plus the elasticity of L in relation to K. The data for S. Korea (PWT) confirm Equation (32).

The constancy of the *Y*/*K* ratio can also be checked more directly by setting its first derivative to zero, as shown as follows:

$$YK' = \left(\frac{Y}{K}\right)' = 0 = \frac{dY}{K} - \left(\frac{Y}{K}\right)\left(\frac{dK}{K}\right) \Rightarrow \frac{Y}{K} = \frac{dY}{K} \Big/ \frac{dK}{K} = \frac{dY}{dK}, \tag{33}$$

Equation (33) tells us that first, the elasticity of K in relation to Y is equal to one; hence the elasticity of Y in relation to K is also equal to one. Second, in the long term, the ratio *Y*/*K* is intrinsically related to the well-known H–D model (see Berg (2001), for a brief exposition and a critic of this model, and Hess (2002) for a more rigorous analysis). In the present study, we do not make any assumptions, such as infinite supply of labor or constancy of the capital-to-labor ratio, which are inherent in the H–D model). To verify our established relationships, we can legitimately take the averages of our sample and check these relationships (in addition to checking with regressions). Thus, an idea of this long-term constant ratio can be obtained by taking the averages for the 64 years of our empirical sample for the numerator and denominator of Equation (33), that is, 0.0136 and 0.0543[7]; hence,

$$\frac{Y}{K} = \frac{dY}{K} \Big/ \frac{dK}{K} = \frac{0.0136}{0.0543} \sim 0.2511. \tag{34}$$

The true value of *Y*/*K* is 0.278 (when it is calculated as per last footnote). Moreover, according to the H–D model (and to some extent, Keynesian due to the assumption of infinite supply of labor), the average yearly growth rate of GDP (*Y*) for our sample (N = 64) should be equal to (Berg 2001; Chiang 1984, Metwally 1995):

$$grY = \frac{dY}{Y} = \frac{dK}{Y} \Big/ \frac{K}{Y} = \frac{s}{k} =\sim \frac{0.195}{3.6} = 0.0543$$

Where s denotes savings (approximated by dK/Y), and k is the K/Y ratio. This result can then be compared with the actual value of *Y*

[7] Given that a small difference exists between the average of Y/K (first version) and the average of Y over the average of K (second version) due to nonsymmetric distributions of data, we adopt the second version for all related ratios, such as Y/K and dK/Y, for easiness of calculation checks for averages.

growth ($grY = dY/Y$), which equals around 4.9% (also calculated as per last footnote, that is, *0.049 = average of dY/average of Y*).

Conversely, if we consider Equation (33) without setting it equal to zero, we obtain a simple differential equation in terms of K' as follows by taking the averages of the corresponding coefficients:

$$\frac{d(\frac{Y}{K})}{dt} + \left(\frac{dK}{K}\right)\left(\frac{Y}{K}\right) = \left(\frac{1}{K}\right)\left(\frac{dY}{dt}\right) \Rightarrow d\left(\frac{Y}{K}\right) + 0.0543\left(\frac{Y}{K}\right) = 0.0136. \quad (35)$$

The solution of this differential equation is (Chiang, 1984):

$$\frac{Y}{K} = y(t) = \left[y\,(0) - \frac{0.0136}{0.0543}\right] e^{-0.0543t} + \frac{0.0136}{0.0543}. \quad (36)$$

Thus, as we take a long period (infinite horizon of time), the solution in Equation (36) tends to Equation (34). Of course, this solution depends on the values of the growth of capital dK/K and the ratio of dY/K. However, these growth rates are interdependent and are thus not expected to change considerably, even in the short run.

In addition, we can have the following relevant relation:

$$YK = \frac{Y}{K} = \frac{dY}{K}\Big/\frac{dK}{K} = \frac{(adLP + bdK + cdL)}{K}\Big/\frac{dK}{K} \Rightarrow a\left(\frac{dLP}{dK}\right) + b + c\left(\frac{dL}{dK}\right) = \frac{dY}{dK}, \quad (37)$$

which is Equation (5), which we have determined earlier. Thus, the relationship between the slopes of the LPF is related to the YK ratio constancy in the long term. The reverse of Y/K, that is, K/Y and hence dK/dY, as per Equation (37), is the well-known investment multiplier (one version). We will also comment on Equation (37) in the next empirical section.

III. Econometric results

The purpose of our econometric estimations is twofold. First, we will be able to check whether our theoretical reasoning is adequate. Second, we will be able to determine some important parameters of economic growth, such as wages, profits, capital, and labor shares. In empirical work, we ought to be vigilant about several problems whenever possible, such as non-stationarity, spurious regressions, heteroscedasticity, and autocorrelation. In this study, we also use alternative estimation methods (*i.e.*, OLS, cointegration, and nonlinear regressions) to make our results as valid and as robust as possible. In many cases, we have stationary data; thus, the OLS regression cannot be spurious. Some of the corresponding graphs confirm the stationarity. Given the novel ideas presented in the previous section, our empirical results ought to be regarded as preliminary for more relevant research to follow by other scholars.

Regarding the data used in our empirical study, for consistency, we utilize the PWT. In any case, PWT has been validated through its popularity in recent years. The period examined is from 1953 to 2017[8].

Table 1 summarizes our regression results (t-stats are shown below the corresponding coefficients). The magnitude, sign, and validity of the regression coefficients must be interpreted correctly.

First, we should estimate the prices of the three contributors to economic growth: capital (K), labor (L), and productivity (LP). Some evidence that the LPF, with the explicit inclusion of LP, can provide us with some preliminary good results; such evidence is important to show. These estimates will depend on how we express the cost and supply function of the economy based on Equations (1), (2), and (13). Regressions (1) and (2) provide us with a first glimpse with all variables expressed as first differences. In Regression (2), the variable dLP is excluded in relation to Regression (1). In Regression (2), the coefficients of dK and dL are larger than the those of Regression (1), which is to be expected because in Regression (1), we have a percentage of the contributions of the coefficients of dK and dL of Regression (2) toward the coefficient of dLP in Regression (1).

Thus, the coefficient 0.178 of dL in Regression (2) becomes 0.086 of dK in Regression (1), and the coefficient 34,130 of dL in Regression (2) becomes 21,775 of dL in Regression (1). Consequently, we obtain an average contribution of dLP in the increment dL in Regression (1), which is equal to 14.76×1046 (1,046 is the actual average for the sample period for dLP) = 15,439 (whereas it was zero in Regression (2)); a contribution of dK equal to 0.086 * 115,500 (115,500 is the

[8] Even more recent data up to 2019 are available, but since they are usually revised annually, our data period is sufficient for robust results.

Table 1
Empirical results for S. Korea

	(1) OLS dY	(2) OLS dY	(3) OLS dY	(4) OLS Y	(5) OLS Y	(6) Coint/n Y	(6') OLS Y	(6'') OLS Y	(7) OLS dY/dK
irr (lag 1)									
b									
Y/K									
LP				6.31				6	
				4.1				4.1	
K/Y									
dLP	14.76		13						
	7.4		6.3						
dLP/dK									6.6
									40.7
K				0.174	0.212	0.213	0.135	0.105	
				18.2	102.4	56	5	4.2	
dK	**0.086**	0.178	0.135						
	5.5	13.5	5.3						
K/L									
L/K									
L				7,284.7	15,548.8	14,807.3	11,474	3,960	
				3.4	21	6.6	7	1.7	
dL	21,775	34,130	23,950						
	5.7	7.2	6.3						
dL/dK									5,433.1
									37
lag of Y or dY			−0.017				0.359	0.328	
			−2.4				2.8	2.8	
Constant	none	none	none	−73429	−103988	−73562.9	−77960	−50900	0.156
				−7.1	−12.9		−6.3	−3.9	11.9
R-squared	0.946	0.9013	0.9529	0.9994	0.9993		0.9993	0.9995	
N	64	64	63	65	65	63	64	64	
lags						2			

Table 1 (cont.)
Empirical results

	(8) Nonlinear	(9) OLS	(10) Nonlinear	(11) Nonlinear	(12) Nonlinear	(13) Nonlinear	(14) Nonlinear	(15) Nonlinear	(16) Nonlinear
	W	**W**	**irr**	**W**	**YK**	**LP**	**labsh**	**YK**	**Y (CDPF)**
irr (lag 1)			−1.223						
			−31.9						
b				0.428	0.416	0.417			
				1,001	215	487			
constant for CDPF				6.67	0.3	0.15			
Y/K			1						
LP	1	0.752							
		39.2							
K/Y							−0.135		
							−69.4		
dLP									
dLP/dK									
K									b = 0.589
									20
dK									
K/L	−0.128	−0.061							
	−118.9	−11.9							
L/K			−3233					1,314.5	
			−23.9					3.5	
L									c = 1.025
									8.6
dL									
dL/dK									
lag of Y or dY									
Constant	none	none					1	0.29	A = 5.9
								24.3	9.3
R-squared	0.9972	0.9994	0.951	0.9983	0.87	0.9759	0.9941	0.1624	0.9996
N	49	49	61	65	65	65	49	65	65
lags									

Note: The number below the coefficient indicates t-stat.

actual average for the sample period for dK) = 9933; and an average contribution of dL equal to 21,775 * 0.31 (0.31 is the actual average for the sample period for dL) = 6750; the total being 15,439 + 9,933 + 6,750 = 32,122, which is to be compared with the average of dY that is equal to 29,010[9].

These results are exploratory. Findings indicate that the capital price (the coefficient of dK) is about 9%, as indicated in Regression (1) and not in Regression (2), which excludes LP. The PWT provides us with a good estimate of the profit rate as a proxy for the capital price, which is the real internal rate of return (*irr*) and is about 11% on average through time. This capital price K can be further checked with Regression (3) of Table 1, which includes the one year lagged dependent variable dY, and for which the coefficients are similar to those in Regression (1). Particularly, the coefficient of dK, 0.135, is the capital price.

The above used averages are calculated for the sample period of the regression; as a property of OLS and other estimation techniques, the averages are legitimate to use for checking the average expectation. The extent to which they approach the true value of average depends on the distribution of the variable concerned (for example, for ratios, the average is not necessarily the same as the quotient of the average of the numerator divided by the average of the denominator, see also a previous footnote); however, we can achieve a good value for our checking calculations.

Instead of first differences, we can use the variables in terms of levels. First, we examine the results of Regressions (5), (6), and (6′), which exclude LP and include only the two factors of production K and L. The coefficient of K in Regressions (5) and (6), as estimated with OLS and cointegration[10] correspondingly, is about 0.21; whereas the price of labor L is about 15,000 US dollars, which is correct in the actual

[9] The check for the yearly forecast for the whole period is also required to appreciate the performance of the model. This checking is done occasionally, but it is not necessary as the forecast of the averages is most certainly sufficient to also expect good results for the whole sample in most cases.

[10] Cointegration uses lags and creates stationary data; thus, any spurious regressions that use data expressed as levels must be checked. In the present study, our cross-checking with various regressions and methods offers some reliability of the estimated coefficients, which can also be checked against the real statistics as per PWT.

data as an average during the sample period[11]. The capital price K is reduced to 0.135 in Regression (6′), which includes the lagged one year Y; thus, it considers all previous years' influence (and autocorrelation econometrically). The previous years' influence is about one-third (0.359) every year.

This same magnitude of the aforementioned influence can be seen in Regression (6″) for the lagged one year Y; for this regression, the LP variable is explicitly included for which the coefficient (millions of labor employed) is 6 and similar to that found in Regression (4). However, the significant difference between Regression (6″) (which is in levels) and Regression (4) (which is in first differences) is the coefficient of capital (its price[12]), which is reduced down to 0.105 in Regression (6″) from 0.174 in Regression (4). Thus, all these regressions indicate that the true value of capital price is about 9%-13%, and the explicit inclusion of the productivity variable LP confirms this capital price.

Subsequently, we comment on the more precise estimates of capital price and wages using the remaining of the modeling presented in the previous section. Regression (8) in Table 1 based on Equation (14) is estimated through a nonlinear method (as per Stata), such that the coefficient of LP is forced to equal one (to have Equation (14) exactly). The results show that the coefficient of variable K/L representing the capital price is estimated to equal 0.128, which necessitates some explanations. Regression (8) indicates that wages (W) are a part of LP; however, it also indicates that "W" is adjusted annually by a fraction of LP to cover profits; this fraction is 0.128 * (K/L). Notably, when we include the one year lagged wages (W) in Regression (8) (not shown in Table 1), the coefficient of K/L becomes 0.091 (as against 0.128). This finding most probably indicates that the capital price of 0.128 in Regression (8) includes depreciation, which cannot be seen explicitly in this regression, but it is included in the lagged W variable. This capital price confirms some of the previous estimates, and it will be further confirmed with the next equations.

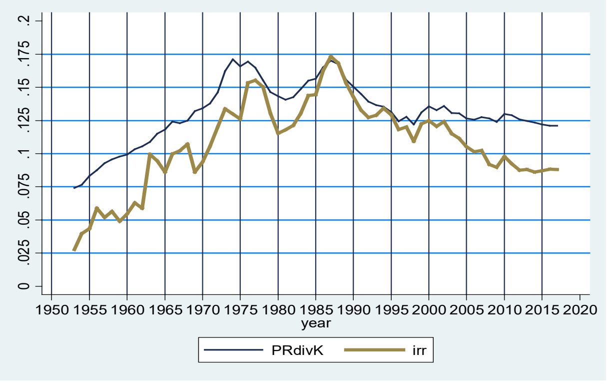
[11] The series of wages is not provided directly by PWT, but we calculate it indirectly through the series of labor share (labsh), income (Y) as GDP, and labor force (L): $W = labsh * (Y/L)$. The average of W for the period examined is 15,553.34

[12] We will define more precisely the meaning of this price further down in this section.

However, we need to further explore Equation (14) at this stage. Regression (9) in Table 1 shows the coefficients of Equation (14) without forcing the coefficient of LP to be equal to 1. This regression tells us that wages are determined in two parts. First, they are about 75% of the yearly LP. Second, they are adjusted by subtracting about 6% of variable K/L(capital-to-labor ratio) from the 75% of LP. This 6% of K/L can be interpreted as the part of the capital price that goes to the investment part of K through the interaction between KL and LP. Thus, the capital price we are determining via our various estimations (say, about 12%–13%) can be split into two parts: about 6% is new yearly investment (actually the growth of capital dK/K is about 6.5%, where dK is investment), and the remaining around 6% is kept as saved profits (part of it may be used to repay loans from banks).

We now check the estimates of the complementary Equation (14) of the previous section, that is, Equation (15). The estimation of this equation is more troublesome because the independent variable Y/K (like the dependent one *irr*) is trend stationary; hence, R-square is expected to be low. This finding is indicated in a regression (not shown in Table 1) in which the coefficient Y/K is forced to be equal to 1 (via a nonlinear method as per Stata program). This regression implies that the proxy for profit rate (*irr*) is primarily related to the ratio Y/K but it is adjusted by a portion of Y/K to consider the contribution of labor L (which is the coefficient of L/K). Given that there exists a strong autocorrelation regarding *irr*, we can include a one year lag of *irr* to obtain Regression (10), which has an extremely high R-square, and the coefficient of L/K is adjusted accordingly.

The symmetrical phenomenon between the determination of wages (as described in the previous paragraphs and with Regressions (8) and (9) especially) and the determination of profit rate (as described by Regression (10) of Table 1) provide us with more precise estimates of the two important components of GDP, that is, wages and profits. To complement our estimation procedure, we also utilize some of Marxian thinking (Piketty 2014; Harvey 2017). Accordingly, the famous profit rate ought to decline in the long run. Figure 3 shows an estimate of such a rate, defined as the ratio of total profits in the economy over the stock of capital. We find that this ratio increased up to the 1980s and then started declining and then stabilized around 12%–13%. A simple regression (not shown in Table 1) of total profits as a function of capital K(without the constant) provides a coefficient of 0.127, which agrees



Note: For the legend, *PRdivK* means the ratio of total profits over the stock of capital *K*; and *irr* (lower curve) means internal rate of return as per PWT.

FIGURE 3

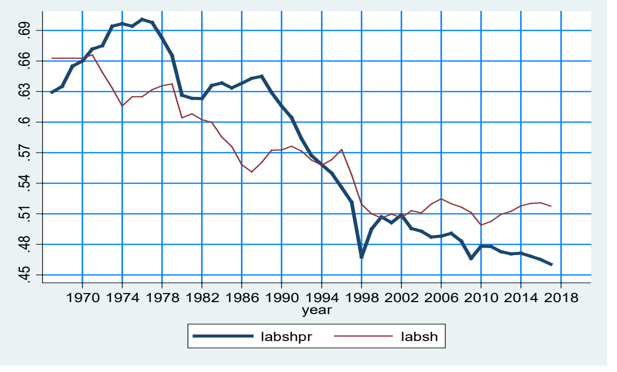
DECLINE OF PROFIT RATE AS TOTAL PROFITS OVER STOCK OF CAPITAL AND irr

with our findings above. In the same graph, we also show the *irr* for comparison, in which the similarity is striking.

We cross-check the split of YK into capital and labor according to Equation (29), given that LP and YK are always split into capital and labor remuneration, as indicated in Regressions (8)–(10). Thus, for the left side part of Equation (29), for the LP ratio, we have profits equal to around 12,000 according to the capital share 0.42 (actual); and for the right side part of Equation (29), for the K/L ratio, we also have around 12,000, which is obtained by applying the profit rate of 12% (the averages once more are for the period 1953–2017).

$$average \quad of \quad \left(\frac{Y}{L}\right) * 0.42 = 28611 * 0.42 \approx 12000 = 0.12 * average \quad of \quad \left(\frac{K}{L}\right) = 0.12 * 100235 \tag{38}$$

We now further check the validity of these results by adopting a CDPF and combining it with an LPF, as shown in the relevant theoretical background in the previous section. One of the main purposes of this checking is the empirical determination of the capital share in output. Although we have a good idea from official statistics of (PWT) the labor share (and hence the capital share as its complement) in the S. Korean economy (see Figure 4), checking on our modeling to validate it will be



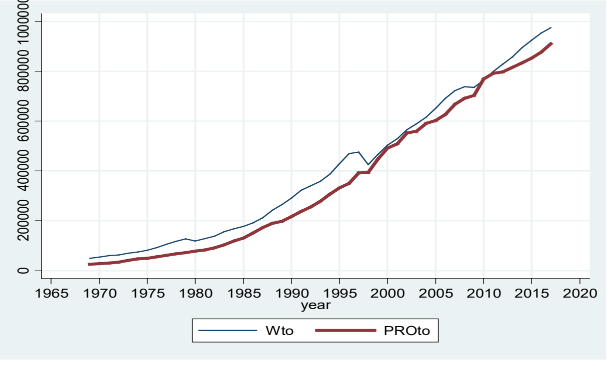
Note: *labshpr* stands for labor share (*labsh*) predicted.

FIGURE 4
LABOR SHARE THROUGH TIME (PREDICTED AND ACTUAL)

worthwhile.

In Figure 4, *labshpr* (predicted *labsh*) is estimated according to Equation (16) and the corresponding Regression (14) in Table 1. The coefficient of K/Y is 0.135, which once more confirms the value for the capital price. The comparison of *labsh* and *labshpr*, as clearly shown in the same graph, indicates that the predicted *labsh* is declining continuously, even after 1999–2000, although the actual *labsh* is flattening out after 1999–2000. This finding implies that after 1999–2000, wages sped up in relation to profits. Symmetrically, before 1999–2000, the predicted *labsh* is higher than the actual *labsh*, which indicates that before 1999–2000, the wages were suppressed downward.

In relation to the capital and labor share, we should also show the calculated total wages and profits (in billion US dollars). As shown in Figure 5, the Asian Crisis of 1997–1999 is a breaking point for the total wage bill, mainly because employment (L) dropped substantially. Most probably, S. Korea never recovered since then in terms of unemployment and related statistics (such as participation in the active labor force). On the contrary, total profits did not seem to have been affected as much as total wage bill. In addition, total wages and total profits stopped growing almost exponentially around 1998–2000 and followed since then a straight line growth as shown in the same graph (and as simple regressions can check). Moreover, notably, during the



Note: *Wto* stands for total wage bill, and *PROto* (lower line) indicates total profit bill.

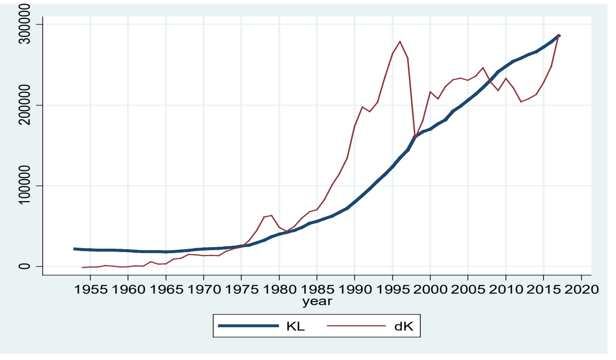
FIGURE 5
TOTAL WAGE AND TOTAL PROFIT BILLS

recent international financial crisis, the total profits started declining, contrary to total wages.

Overall, we need to see in detail this structural break around that date, something which is out of the scope of this study. Nevertheless, Figure 6 sheds more light to this break. Thus, in Figure 6, we can see that investment (*dK*) slowed down considerably during and after the Asian Crisis. The break can also be seen but not as clearly as for *dK* in the *K*/*L* series. Related to all this and in particular to investment is the growth rate of GDP (*Y*) as a reaction to the growth of capital (*K*), which is shown in Figure 7 where we can clearly observe that as the capital growth slowed down considerably after 1998, so did the GDP growth. On average, the GDP growth was 6.8% per annum, and the capital growth was 6.6% per annum for the whole sample period.

Capital share has also been the object of estimation by other scholars, but indirectly either through actual data (although contradicting the PWT data as we note in the next footnote) or through the convergence issue of national economies and usually adopting the CRS assumption and Solow model (see for example a good summary in Chapter 3[13] of

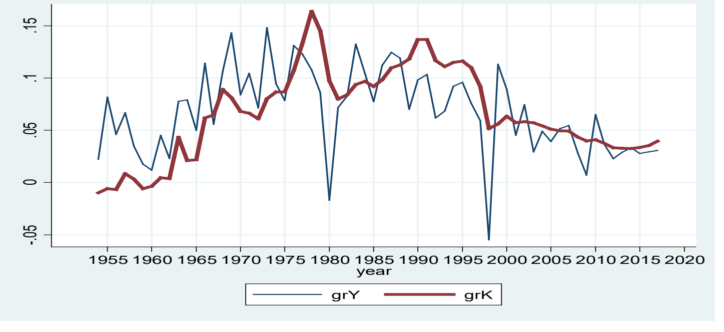
[13] Acemoglu (2009, p. 57) also showed a graph that depicts the capital and labor share of the US economy. However, it does not mention the source of the data corresponding to this graph, which is unfortunate because the value



Note: *KL* stands for the ratio *K*/*L* and *dK* stands for yearly increment in capital *K* or investment.

Figure 6

Investment and capital-to-labor ratio



Note: *grY* stands for growth in GDP, and *grK* (the smoother curve) stands for growth in capital stock.

Figure 7

Growth rates of GDP and capital

for capital share shown in his graph and also used to check some relevant regressions is around 1/3. However, this value significantly contradicts the labor share (and hence the capital share) as indicated in the PWT, which is about 0.40, unless the depreciation rate is already subtracted in Acemoglu's graph; nevertheless, there is no agreement with the well-known PWT official series. Notably, other books indicate such value for capital share (e.g., 0.30 by Abel et al. (2011, p. 60)) without indicating the appropriate sources. Many scholars seem to have accepted such a value without question. However, in other studies (Aum and Shin, 2020), the labor share agrees with that of PWT data. We follow a different approach whereby we cross check all our estimates in many ways.

Acemoglu's work (2009)). Here, we will attempt to make more direct estimates based on our modeling of the LPF and the combination of the LPF with the CDPF.

Effectively, we use the nonlinear method again to estimate Equations (26)–(28), and the results are shown in Regressions (11)–(13) in Table 1, respectively. We should have consistency in all the three equations regarding the parameter *b*, the capital share. Given that we have a good idea to what *b* actually is, we initially estimate Equation (26) (copied from the previous section for convenience, see below) which provides the values of m = 1/0.0717 = 14 and b = 0.367 (and R-square equaling 0.9993) via the nonlinear estimation process (not shown in Table 1). However, we can search for a value *m*, such that we obtain *b* = 0.42 (the true value) instead of 0.367. This value of *m* is 1/0.15 = 6.67, which is shown in Regression (11)[14] with an R-square that is still as high as the previous when parameters *m* and *b* are estimated simultaneously (0.9983 instead of 0.9993).

$$A = f(W) = m''m'K^bL^{1-b} = mK^bL^{1-b}, \text{ copied from previous section} \quad (26)$$

Then, we use this estimate of *m* = 1/0.15 to obtain an estimate of constants α_1 and α_2, such that we can estimate Equations (27) and (28). The estimation of α_1 and α_2 is briefly as follows (from Equation (27), which is shown below again to obtain Equation (39)):

$$f\left(\frac{Y}{K}\right) = \alpha_1\left(\frac{Y}{K}\right) = mAbK^{b-1}L^{1-b} = mbK^bL^{1-b}K^{b-1}L^{1-b} = mbK^{2b-1}L^{2-2b}, \text{ copied} \quad (27)$$

$$W\Big/\frac{Y}{K} = \frac{W}{0.32} = \frac{\alpha_1}{bK^{b-1}L^{1-b}} \Rightarrow \alpha_1 \approx \frac{15500 * 000443}{0.32} \approx 21.5, \quad (39)$$

where the numbers shown are the averages[15] for the regression period of 1953–2017 and with an assumed *b* = 0.42(which is though confirmed in the process, see below too). Thus, with *m* = 1/0.15 and α_1 = 21.15, we can have an estimation of Equation (27) by using the nonlinear regression as per Stata, as shown as follows:

[14] The nonlinear command in Stata program is nl(W = K^({b = 0.4}) * L^((1 − {b = 0.4}))/(0.15)).

[15] As we noted earlier, we can use these averages because OLS estimation is centered on such averages (*e.g.,* for a variance, we have x_t − average of x_t).

$$nl\left(YK = 0.30 * (\{b = 0.4\}) * K^{(2*\{b=0.4\}-1)} * L^{(2*(1-\{b=0.4\}))}\right), \tag{40}$$

where the constant 0.30 (rounded off) in Equation (40) is calculated as follows:

$$0.31 = \frac{m}{\alpha_1} = \frac{6.67}{21.5}.$$

This regression (Equation (40)) provides b = 0.416. Note that without this precise value of 0.30 as above, the nonlinear estimation will not have provided the correct value for *b*.

Similarly, from Equation (28), we obtain the following:

$$W\bigg/\frac{Y}{L} = \frac{W}{28600} = \frac{\alpha_2}{(1-b)\,K^b L^{1-b}} \Rightarrow \alpha_2 \approx \frac{15500 * 82.94}{28600} \approx 44.9. \tag{41}$$

Again, the numbers shown are the averages for the regression period of 1953–2017 and with $b = 0.42$. Thus, with $m = 1/0.15$ and $\alpha_2 = 44.9$ and thus $0.15 = m/\alpha_2 = 6.67/44.9$, we can have an estimation of Equation (28) by nonlinearly regressing, as shown as follows:

$$0.15 = \frac{m}{\alpha_2} = \frac{6.67}{44.9},. \tag{42}$$

The above regression provides $b = 0.417$. Without the precise estimates of constants α_1 and α_2, as well as the constants 0.15 and 0.30, the way only described, we could not have found in the above regressions the correct value for *b*.

Consequently, the share of capital in GDP is confirmed through our theoretical and empirical modeling to be around 0.415, which is true for the S. Korean economy in the period examined. Certainly, this estimation depends on the estimates of constants 0.30 and 0.15 in Equations (40) and (42), respectively, and the calculation of the constants in the regression based on Equation (26). Hence, we maximize our degrees of freedom to obtain an indirect estimate of the capital share *b*.

We estimate the CDPF in several ways depending on how many parameters we estimate simultaneously in the nonlinear regression. If only *A* is estimated and having fixed *b* as equal to 0.42 and forcing CRS in the CDPF (hence $c = 1 - b$ in Equation (43)), then *A* equals 317.1.

$$Y = AK^bL^c \tag{43}$$

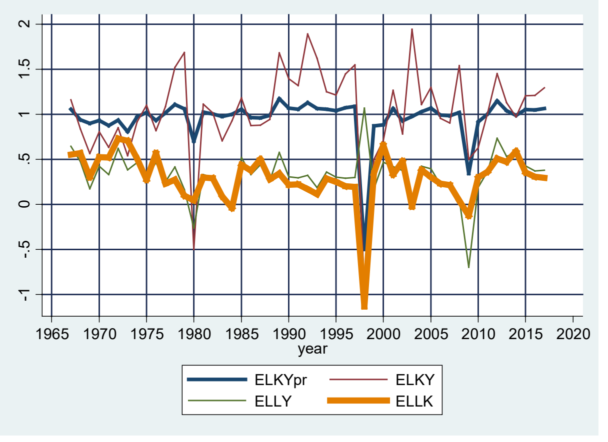
Table 1 only shows the detailed results for the CDPF (Equation (43)), for which all three parameters *A*, *b*, and *c* are simultaneously estimated through the nonlinear method (Regression (16)). In Regression (16), the CDPF does not exhibit CRS. We also estimate Equation (43) when only *A* and *c* are simultaneously estimated (again no CRS). Moreover, when *A* and *b* are simultaneously estimated, but in which CRS is forced, in this case *b* equals 0.787, which is different from the actual *b* equaling around 0.42. We also estimate Equation (43) in log terms, for which *b* is estimated to equal 0.416 (which is the correct value), and *c* equals 1.66, with log *A* equaling 2.42; again, the CDPF exhibits IRS (but this time *b* being correct).

Finally, we investigate the reactions of the main variables to an increment of K, that is, investment *dK*. Thus, we want to see the evolution of *dY/dK*[16] (a version of investment multiplier), as shown in Regression (7) in Table 1 based on Equations (5) and (37). About 22.5% of the ratio *dY/dK* is the contribution of *dLP/dK*, 31.2% of *dL/dK* and 46.2% of K. Regression (7) is an estimation of *Y/K* with slopes and can be used for forecasting purposes. The econometric estimation of *dY/dK* also provides an estimation of the investment multiplier, as mentioned in the previous section.

Finally, we confirm Equation (32) regarding the constancy of *Y/K* as seen through some relevant elasticities. Figure 8 shows the actual and predicted elasticity ELKY of K in relation to Y (the predicted elasticity is based on Equation (17)). Both of them hover around one, as expected. Conversely, elasticities ELLY (*L* in relation to *Y*) and ELLK (*L* in relation to *K*) hover around the value 0.3 (actual data). Given that ELKY is equal to one, ELLY should be equal to ELLK, which is true.

We now confirm some of the above results by estimating panel data (cross-sectional with time series). The source of data is again the PWT for 2019 (the last one available). The source contains 183 countries; however for some of them, not all data are available, especially employment statistics. Thus, after elimination, we have 131 countries

[16] By dividing all parts of Equation (4) by $dK = I$, we eliminate heteroscedasticity problems. This elimination is checked with an estimation without the division by $dK = I$, which presents serious problems of heteroscedasticity.



Note: We eliminate the period of 1953–1966 from the graph because this period exhibits some wider fluctuations, which will affect the clarity of the graph. We also correct one datum for ELKYpr (ELKY predicted) for 1998 as it is exceptionally negative and two data for ELKY for 2 years for the same reason. The corrections and adjustments do not alter the messages we obtain from the graph as per text. The acronym ELKY stands for elasticity of *K* in relation to *Y*.

Figure 8
Elasticity of K in relation to Y

in our sample, as shown in Table 4. Once more, we use the variables as shown in Table 2. We should also make some important remarks from the outset regarding the use of panel country data.

We cover almost all the world (the excluded countries are the smallest ones and with incomplete available data (*e.g.,* Aruba or Saint Kitts)), the random effects panel data model is not appropriate to use here (but is nevertheless cross-checked and provides almost the same results as the fixed effects (FE) model). In addition, the regression results for all countries in the sample may cover important differences between various countries. Hence, only a detailed examination for each country separately might allow us a detailed and rigorous comparison. However, this examination exceeds substantially the purpose (and length) of this study. Nevertheless, we will add whenever appropriate some comments on the important differences between nations (*e.g.,* according to the intercepts of the FE model).

TABLE 2
VARIABLES USED IN THE PANEL DATA

PWT code	Variable meaning	Our symbol
rgdpna	GDP at constant prices	Y
rnna	K at constant prices	K
emp	L in millions	L
labsh	Labor share in GDP	labsh
irr	Internal rate of return	irr
Wages are not explicitly included in PWT	Real wages	W = labsh*Y/L

Table 3 presents the panel data estimations.

In Column (1), we have the FE regression for the dependent variable *dY* as a function of *dK*, *dL*, and *dLP*, which is equivalent to Equation (1) of Table 1 for S. Korea. In Column (2), the estimation based on MLE method calculates the panel coefficients as in previous Column (1) but without the constant. We also estimate the individual intercepts for each country due to the FE version (not shown in Table 3). Most of them are not significantly different from zero, except for about 10 nations, mainly including big countries such as the USA, China, India, Japan, Russia, and Germany and some resource countries such as Kuwait and Saudi Arabia. In Column (3) of Table 3 the panel estimation is shown as per previous two columns, but the *dLP* explanatory variable is excluded. In Columns (4) and (5), the panel estimation of *Y* as a function of *K* and *L* takes place; the difference between Columns (4) and (5) is the lagged one year *K* added in the regression. In the five panel data estimations, the coefficient of *K* or *dK*, which indicates the price of K, is consistently equal to around 0.13 and is similar to what is found for S. Korea (Table 1). All other coefficients are also valid and confirmed by the extremely high R-square.

Here, we present the estimations of profits and wages. For the sake of panel data regressions (thus making the calculations of relevant panel dummies more direct to what we want to show), we initially examine the profits either as a sum of money (variable *LPmW* being the difference between LP and W in Equation (44)) or as a profit rate (variable *proK* in Equation 44). These variables are deduced as follows from the

Table 3

Panel data estimations

Equation	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Model	panel FE	panel MLE	panel FE	panel FE	panel FE	panel FE	panel FE	panel FE	panel FE
Variable		without constant				with dummies	with dummies	All	Germany
Dependent	**dY**	**dY**	**dY**	**Y**	**Y**	**proK**	**LPmW**	**logY**	**logY**
time						−0.00076			
						−13.8			
logK								0.723	0.784
								131.3	52.7
logL								0.312	0.7
								30	4.7
dLP	1.62	1.62							
	15	15.2							
K				0.217	**0.125**				
				259	7.7				
dK	**0.134**	**0.139**	**0.135**						
	74	79.3	73.5				**0.0777**		
K/L							51.7		
L				4993.1	5323.7				
				38.7	37.2				
dL	6,206	7,487.5	6167.5						
	11.7	15.2	11.4						
lag of K					0.096				
					5.6				
Constant	3,494	none	4,091.9	−66,724.2	−74,070.6	0.1327	1782.1	1.77	−0.625
	7.7		8.9	−22.6	−23	11.3	1.2	30.5	−1.7
R-sq within	0.45		0.434	0.945	0.945			0.916	0.994
R-sq between	0.92		0.924	0.94	0.938			0.933	
R-sq overall	0.645		0.635	0.947	0.946	0.666	0.821	0.936	0.994
N obs	7,471	7,471	7,471	7,602	7,471	6,481	6,481	7,602	70
N groups	131	131	131	131	131	111	111	131	1

fundamental Equation (14):

$$W = \frac{Y}{L} - \beta\left(\frac{K}{L}\right) \Rightarrow \frac{Y}{L} - W = (LPmW) = \beta\left(\frac{K}{L}\right)$$
$$\Rightarrow \beta = \frac{\frac{Y}{L} - W}{\frac{K}{L}} = \frac{Y - WL}{K} = \frac{profits}{K} = proK. \tag{44}$$

In Column (6) of Table 3, we estimate the panel equation $proK = s * time + cons\tan t$. The constant is equal to 0.1327 which once more indicates the capital price K similar to previous estimations. This constant is an average over the whole set of panel data, but if we estimate this constant on an individual basis, it varies from country to country, as expected and similarly for the panel dummies (not shown in Table 3). Thus, for S. Korea, the constant is 0.087 (with t-stat = 20.8). For the data we use for Table 1, the constant is 0.1177, the difference being the updating of the 2019 PWT against the 2017 PWT. For the USA, the constant is 0.0836, and 0.1245 for Japan (but with negative *s*, slope of time variable). Moreover, if we apply Equation (14) to the USA, Japan, and other countries, then the results are similar (with some small differences at times) to those obtained here with panel data.

In Column (7) of Table 3, we show the panel data regression of the variable profits (in terms of money) as per variable *LPmW* of Equation (44), with explicit dummies for each country shown in Table 4. The coefficient of β of the explanatory variable K/L for the whole set of panel data is 0.0777 (as per regression). Regarding the individual country performance as per dummies in Table 4, we initially notice that for S. Korea, the profits are neither higher nor lower than the average suggested by the panel data regression (the t-stat of the dummy is extremely small). Thus, at the same time, the wages are rather normal for S. Korea, as this level of being "normal" is suggested by the panel data regression.

Second, as shown in Table 4, the countries that have rather abnormally high profits (the dummies are highly and significantly positive)[17] are mostly the resource rich ones, such as Kuwait and Saudi Arabia (with some exceptions such as Ireland). Note that relatively high

[17] The value of these dummies should be added to the overall constant of the panel regression (which is 1,782.1 as per Table 3) to obtain precise figures of the individual FE. However, given that the present work is a comparative study, this is not relevant here.

profits may also imply high rent income from buildings, given that "profts" include this rent. Conversely, when the dummies are negative (thereby expressing high wages (*W*) in relation to *LP*), the wages are rather on the high area of the spectrum in the developed countries (with some exceptions such as the USA).

Finally, the performance of CDPF with the panel data is shown for the whole sample in Equation (8) of Table 3. The elasticities in relation to *K* and *L* sum up to approximately 1, although the coefficient of *K* is extremely high (0.723), as compared with the actual capital share in GDP for the whole world which is 42.5%. Thus, on a global view, the world has CRS, a result which is not surprising, given that many individual country results may cancel out. Effectively for some countries such as Germany (also shown in Table 3), S. Korea, the Netherlands, and Japan, we have IRS, whereas for especially big countries such as China and India, we have CRS; and for some other countries (not many), we even have decreasing returns to scale.

However, the coefficients usually have the wrong magnitude (such as in the case of Germany as mentioned previously). Consequently, we face an important issue here. On the one hand, the coefficients of a CDPF for *K* and *L* must equal 1 by definition when added up (since the capital share is 1 minus the labor share), which leads us to CRS. On the other hand, if we recognize that in some countries we might have IRS (such as in most developed nations), then we may use Equation (30), which incorporates IRS and still forces the coefficient of K to be equal to 0.425 (hence, the labor share is 57.5%, and the addition of the two elasticities may add up to 1 as required in the context of a CDPF). To perform some regressions for other countries by using Equations (30), (20), and (28), we need to initially establish the appropriate parameters as we did for the case of S. Korea, which will, however, add considerably to the present study (and is thus beyond its scope). Perhaps future scholars may do a comparative study between many nations and find the relevant conclusions according to the methodology suggested here.

All the above results confirm the following points, thereby making the relevant estimates valid and robust. In addition, our theoretical developments in Section II are also confirmed.

(a) Parameter *b* has the well-known meaning of capital (*K*) share in GDP, which is approximately 42%. The PWT provides an estimate of this capital share through labor share (*labsh*); the average of *labsh* equals around 58.5%.

TABLE 4

PROFIT PANEL DATA WITH COUNTRY DUMMIES (AS PER COLUMN 7 OF TABLE 3); DEPENDENT VARIABLE IS *LPmW*

Country	Coef.	t-stat	Country	Coef.	t-stat	Country	Coef.	t-stat
Albania	0		Hong Kong	−7,820.28	−3.73	Peru	2,312.128	1.17
UAE	0		Croatia	−5,189.41	−2.09	Philip/es	2,285.753	1.15
Argentina	12,356	6.24	Hungary	−1,560.84	−0.73	Poland	6,084.161	2.85
Armenia	1,877.8	0.76	Indonesia	299.6312	0.15	Portugal	−12,863.1	−6.39
Australia	1,520.5	0.76	India	−1,149.32	−0.58	Paraguay	3,319.626	1.67
Austria	6,483.6	3.19	Ireland	13,334.43	6.61	Qatar	10,7822.6	48.32
Azerbaijan	8,507.6	3.45	Iran	12,624.36	6.26	Romania	5,876.523	2.87
Belgium	13,409	6.48	Iraq	23,730.55	11.11	Russian F.	−3,075.55	−1.24
Burkina F.	712.91	0.35	Iceland	−9,951.95	−4.93	Rwanda	−1,403.78	−0.68
Bangladesh	0		Israel	2,872.718	1.45	Sa/i Ar/bia	87,573.39	40.02
Bulgaria	7,673.9	3.59	Italy	−6,478.79	−3.15	Sudan	−232.562	−0.11
Bahrain	44,337	19.8	Jamaica	−2,314.42	−1.16	Senegal	101.3505	0.05
Bosnia Her	4,778.9	1.94	Jordan	13,398.88	6.69	Singapore	12,812.34	6.18
Belarus	3,260.7	1.32	Japan	−477.805	−0.24	Serbia	−6,666.42	−2.7
Bolivia	1,853.6	0.94	Kazakhstan	9,563.517	3.88	Slovakia	−3,150.65	−1.27
Brazil	1,783.3	0.9	Kenya	−748.685	−0.38	Slovenia	−17,492.7	−6.93
Botswana	7,274.8	3.56	Kyrgyzstan	827.1405	0.34	Sweden	−6,632.57	−3.26
Canada	2,905.8	1.45	Cambodia	0		Syria	0	
Switzerland	10,882	5.23	Korea Rep.	1,403.094	0.7	Chad	−561.028	−0.27
Chile	5,709.7	2.88	Kuwait	105,435	48.88	Thailand	−3,154.21	−1.59
China	192.57	−0.1	Lebanon	7,535.658	3.47	Tajiki/tan	−13,812.5	−5.58
Cote d' Iv	1,560.5	0.76	Sri Lanka	3,751.403	1.89	Turkm/tan	0	
Cameroon	374.53	0.18	Lithuania	3,071.62	1.24	Trin. & To.	14,817.97	7.49
Congo DR	0		Luxemburg	−11,118.7	−5.18	Tunisia	2,953.242	1.44
Congo	0		Latvia	−15,520.6	−6.16	Turkey	10,519.21	5.32
Colombia	3,978.4	2.01	Morocco	−704.662	−0.36	Taiwan	−2,977.24	−1.5
Costa Rica	4,653.6	2.35	Moldava	−287.469	−0.12	Tanzania	−719.375	−0.35
Cyprus	15,733	7.65	Madagascar	0		Uganda	0	
Czechia	8,690.7	3.45	Mexico	8,663.386	4.37	Ukraine	−20,554.1	−8.19
Germany	3,770.1	1.88	N. Maced.	1,246.645	0.5	Uruguay	2,780.655	1.41
Denmark	−4,622	2.29	Mali	0		USA	5,812.069	2.9
Dominican	3,423.5	1.72	Malta	5,035.708	2.51	Uzbek/tan	4,343.598	1.76
Algeria	0		Myanmar	0		Venez/la	−2,384.81	−1.2
Ecuador	3,117.5	1.57	Mont/gro	0		Vietnam	0	
Egypt	10,511	5.31	Moz/que	−1,044.46	−0.51	Yemen	0	
Spain	5,109.9	2.54	Mauritius	5,074.571	2.57	S. Africa	2,367.381	1.2
Estonia	2,016.7	0.81	Malawai	0		Zambia	−4,823.12	−2.4
Ethiopia	0		Malaysia	6,581.892	3.27	Zimbabwe	−1,274.33	−0.64
Finland	1,896.7	0.95	Namibia	3,980.597	1.95			
France	9,575.9	−4.7	Niger	−2,283.14	−1.11			
Gabon	15,258	7.43	Nigeria	−2,132.25	−1.08			
UK	315.49	0.16	Nether/ds	−7,043.27	−3.48			
Georgia	351.31	0.14	Norway	7,952.771	3.92			
Ghana	0		New Zeal.	6,817.82	3.44			
Greece	7,493.1	3.68	Oman	37,064.74	17.14			
Guatemala	1,030.5	0.52	Pakistan	0				

(b) According to Equation (17), the profit share of the economy should be equal to $\Pi \quad share = 1 - labor \quad share = 1 - \left(1 - \beta \frac{K}{Y}\right) = \beta \left(\frac{K}{Y}\right)$. Thus, with an average value of K/Y equaling 3.6 in the S. Korean economy, and a profit rate equaling 0.12 (from previous discussed estimations), we obtain an approximation of the capital share equaling 3.6 × 0.12 = 0.432 or about 43%.

(c) We arrive at these conclusions by considering and including explicitly in our analysis the LP, LPF, and CDPF. This feature is also a novelty of this study.

(d) Given that we use LP explicitly, we cannot transform the variables into logs; hence, we use a nonlinear technique.

(e) Technically, with the estimation of the CDPF, we can “force” the data to behave similar to CRS if we want to do that. According to the evidence provided here, when we explicitly include A as a variable involving factors K and L (and not as a constant or as a function of time, as many scholars have done), the PF should have IRS.

IV. Conclusions and policies

We summarize the answers provided to the questions in the Introduction as follows.

(i) Can we use LPF? The answer is yes, as the entire study has demonstrated.

(ii) Can we combine LPF with CDPF? Similarly, we show that the answer is yes.

(iii) What is the capital price and the labor price? Our theoretical and empirical findings show that the capital price is about 13% and the wages about 15,000 US dollars on average during the sample period of 1953–2017.

(iv) What is the split between profit and wage shares in the economy? Our theoretical and empirical findings show that the profit share is about 42% of the GDP.

(v) Particularly, what does it mean and how is it explained that the wage share is rather declining? We provide some theoretical and empirical evidence that this decline is not a surprising result given the free development of the S. Korean economy. Thus, as we further explain in the long term, wage share remains constant in S. Korea, and the declining share is mainly due to the increasing capital-to-GDP ratio (K/Y).

(vi) Do we have CRS or IRS? We show that theoretically and empirically, the correct answer is IRS.

(vii) Do we have a balanced growth whereby some economic variables or their combinations are constant or nearly constant through time? We show that theoretically and empirically, the S. Korean economy has been growing in a balanced manner (the same conclusion can be expected for any other economy). Thus, several equations and regressions demonstrate this issue (*e.g.*, the constancy of the Y/K ratio and the constancy of some elasticities).

In more details, in the first subsection of the theoretical section, we establish the LPF based on K, L, and A. We also establish the considerably important basic equations of what the marginal products of the three contributors to GDP are. Thus, the price of labor is a function of the Y/L ratio (LP); the capital price is a function of the Y/K ratio; and the price of LP (as a proxy for A) is a function of labor force (L). In the second subsection of Section II, we establish the following fundamental equations based on the LPF, which adequately describe the wages and profits.

Wages: $W(t) = \frac{Y}{L} - \beta \frac{K}{L}$

Profits: $i(t) = \frac{Y}{K} - c \frac{L}{K}$

Labor share: $\left(1 - \beta \frac{K}{Y}\right)$

Profits share: $\beta \left(\frac{K}{Y}\right)$

In the third subsection of Section II, we combine the neoclassical CDPF with our LPF to establish precise MPs for productivity A, capital K, and L(see appropriate equations in the text), which can then be econometrically estimated. To this effect, we propose that productivity is a function of wages, $A = f(W)$, and, consequently, that our proposed CDPF is $Y = mK^{2b}L^{2(1-b)}$, which has IRS and not CRS. In the fourth subsection of Section II, we establish a detailed theoretical analysis about the long-term constancy of the ratio Y/K(or capital productivity) in three ways. First, we establish the formula

$$E(K,Y) = \frac{[1+E(L,Y)]}{[1+E(L,K)]},$$

which, as a consequence of this constancy, links the three elasticities as seen in this formula. Second, we relate this Y/K constancy to the well-known H–D growth model and its validity. Third, we cross-check this constancy through the solution of a relevant differential equation.

In the third section and following the above subsections of the theoretical section, we initially provide some evidence of the validity of the LPF by regressing various forms of this function, with confirmatory results regarding the MPs and hence the coefficients of this LPF Table 1 summarizes the empirical results for S. Korea. Tables 3 and 4 summarize the results for 131 countries panel data. Then, we further estimate the wage and capital prices, as well as the wage and profit shares in the economy according to the equations we establish in the second subsection of Section II. Then, we estimate the MPs (prices of capital and wages) via the established equations of the combination of the LPF and CDPF. At the same time, we provide some key graphs affirming the empirical results. Finally, we provide some econometric evidence as to the constancy of the Y/K ratio according to the theoretical findings.

Thus, we can summarize some of the numerical findings. The labor share is confirmed to be about 42%, and, together, the profit rate is confirmed to be about 13%. The two variables, share of capital and profit rate, are interdependent; hence, we have determined their theoretical interrelation and their empirical estimations. Thus, a 13% capital price entails a 42% capital share based on the capital to ratio average of about 3.25, according to the equation we established in theory (0.42 = 0.13 * 3.25)..

Conversely, the labor share has been declining recently from about 58% on average to almost 50%–51% by the end of 2010 (see also Figure 4, which shows that the labor share has remained constant at about 51% during the period of 2000–2017). This decline is not due to declining wages but to the declining of the Y/K ratio or its inverse, that is, the increasing capital to GDP ratio (K/Y). Thus, according to our formula, the labor share by the end of 2010 should be:

$\left(1-\beta\frac{K}{Y}\right)=1-0.13*3.8=0.506$ or 50.6%, which is true in the actual data.

In addition, our established relations show that the wage per employee in the economy should be equal to the LP multiplied by the labor share, hence, by the middle of 2000–2017, the wage should be equal to: $W=labsh*\left(\frac{Y}{L}\right)=0.506*60000=30360,$ which is also true in the actual data. Then, the question arises as to why the *K*/*Y* has been slowly rising, which is a complex question that requires a separate study to answer it properly. This increase or the equivalent of a slowly decreasing *Y*/*K* is probably due to the decreasing efficiency of the added capital (investment) to generate income (GDP) mainly because of the increasing share of services in the economy.

The capital price is about 13% per year, which can be regarded as the risk premium for investments. Is it abnormally high? To answer such a question, we need to relate it to the high growth of GDP in S. Korea, which, on average, has been about 6.5% during the period of 1954–2017. Hence, a "high" capital price (and hence profit rate) is intrinsically related to a high rate of economic growth. Note that the difference between the rate of capital price of about 13% and the investment (or capital growth) or about 6.5% can be considered mainly repaying business loans.

Overall, we cannot see any sign in our theoretical and empirical results that profits have been increasing to the detriment of wages in the S. Korean economy. Our analysis and various graphs (for example Figures 1 and 5) are a testimony of this observation. Rather, we ought to examine in more detail as to why a structural break existed around 1999–2000, which led to considerably smaller growth increases in investments and hence GDP (see Figures 6 and 7). Was that due to smaller returns to capital? Some evidence proves it (with *irr*, the internal rate of return, and as Figure 3 shows). However, the capital price remains constant to about 13% (the profit rate is not necessarily the same as the capital price). Further research must be conducted about all these, which includes long-term constancy of some important ratios, such as *Y*/*K* (see Figure 2) and elasticities (see Figure 8), as we have analyzed in detail.

Moreover, one might also ask why the slowing down in economic growth took place immediately after the Financial Asian Crisis of the late 1990s and the ensuing radical changes in chaebols' governance and transparency. Similar to many other typical advanced countries, S.

Korea also experienced "de-industrialization" and shifted to the services sector (Jung (2016, p. 274). In this respect, see also Sanidas and Park (2011). Consequently, to further understand the results presented here, we ought to understand more the structural changes of the S. Korean economy, which took place after the mid-1990s and especially after the Asian Crisis of 1997–1999.

In addition, economic growth and related development are complex phenomena; thus, several points have not been treated in this study, such as the role of institutions and politics (Sanidas, 2017), big businesses (Lee *et al.* 2013), (mis)management of the economy (Sanidas 2014), transaction costs and the black box (Sanidas 2006), and organizational innovations (Sanidas 2005). Moreover, links between S. Korean and international macroeconomic fluctuations (Kim 2011) should be considered. For example, Kim (2011 p. 14) said that "The real exchange rate responses to demand shocks are also larger after than before the Asian financial crisis," thereby confirming our overall suggestion that the Asian crisis is an important structural break as related to our findings and it ought to be considered. The influence of technology also plays an important role in the analysis (Sanidas 2004).

Furthermore, Kim (2016) indicated that competition with other developed nations such as Japan plays a significant role after 2000 and following the Financial Asian Crisis of 1997. Furthermore, "… Most of big businesses from Korea, with their specialization in more capital-intensive sectors, started internationalization only from the mid-1990s… Since this period, these overseas factories of big business have become a public concern in terms of the possible hollowing out of Korean industry…" (Jung 2016, p. 273-74).

We can thus see that the results analyzed and shown in this study are inherently limited to be stylized and hence exclude so many other issues mentioned. However, they may still be valid in this more general context. S. Korea has achieved a remarkable catch-up in relation to other developed countries. In our study, we emphasize some macro-economic relations (based on the LPF methodology) interwoven with micro-economic variables, such as wages and profits, which confirm this catch-up. Then, leaving this wage–profit debate out of political expediencies and base it more on hard evidence is important, as presented in the current study despite the necessary shortcomings.

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