Cheap-Talk Disclosure of Negative Information and Risk-Averse Buyers

Dmitry Shapiro and Seung Huh

In this paper, we study the incentives of low-quality sellers to separate them from high-quality sellers. We consider a framework with asymmetric quality information where the only way to communicate quality is via cheap-talk messages. In this framework, any separating strategy pursued by high-quality sellers can be imitated costlessly by low-quality sellers. We show that in the duopoly setting with risk-averse buyers, equilibria exist, where low-quality sellers voluntarily disclose negative information about their products. If the seller is a monopolist or buyers are risk-neutral, such equilibria do not exist.

Keywords: Negative information, Risk aversion, Cheap talk, Product differentiation

JEL Classification: D21, D81, L15, M31

I. Introduction

The setting where the product quality is the seller's private information has been studied extensively in the literature with main focus on the tension between consumers who want more quality information and low-quality sellers who would like to hide it (Dranove

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and Jin 2010). In the recent "*Dieselgate scandal*", for example, Germany's largest automobile manufacturer, Volkswagen, was found to have cheated on emissions tests for almost half a million diesel models to hide the fact that its vehicles' NO_x emission was much higher than what the company had claimed (Atiyeh 2016). Going beyond anecdotal evidence, that negative information can hurt sellers, and, therefore, low-quality sellers might have incentives to hide it, is well-established in academic literature (for a detailed review see, *e.g.*, Berger *et al.* 2010).

However, we still observe sellers voluntarily revealing negative aspects of their products or services. For example, Avis, a rental car company, once ran a bold advertising campaign with the statement, "Avis is only No. 2." Although its campaign explicitly confessed that it was not as good as Hertz, the leading company in the rental car industry at the time, this honest message appealed to many customers and enabled Avis to turn a \$3.2m loss into a profit of \$1.2m.[1] Ironically, Volkswagen, the same manufacturer responsible for the Dieselgate scandal, ran a famous "Lemon" advertising campaign in 1960, in which they claimed some of their cars were lemons because "the chrome strip on the glove compartment is blemished." Two-sided advertisement wherein sellers include negative information in their quality claims alongside positive information is a well-known marketing practice among sellers (Crowley and Hoyer 1994; Eisend 2006, 2007; Settle and Golden 1974). Finally, in consumer-to-consumer online marketplaces, such as Craigslist, sellers can be commonly observed to describe weaknesses of their listed products via cheap-talk messages, such as "*the product is in fair condition*", and via verifiable information, such as pictures of specific damages and scratches that are otherwise indiscernible.

Theoretical literature, while not numerous, has explored the phenomenon of revealing negative information and showed that even in settings where no reputation can be built and no costly signals, such as warranties, are available, equilibria can exist even where low-quality sellers choose to *reveal* rather than *hide* negative quality information. Board (2009) and Guo and Zhao (2009) showed that in the framework with risk-neutral buyers where information disclosure is credible and verifiable, low-quality sellers may disclose their types if

[1] https://www.campaignlive.com/article/history-advertising-no-177-robert-townsends-all-staff-memo/1403089

a loss from a lower perceived value is smaller than the gain from the decreased competition with high-quality sellers. Revealing negative information is also possible in settings where no credible and verifiable way to disclose quality information exists but market frictions, such as search or matching frictions, can be observed. Kim (2012) and Gardete (2013) have shown that low-quality sellers may want to reveal their types through cheap-talk messages because it can either reduce the search costs of buyers or reduce the competition intensity among sellers, thereby increasing sellers' profit. Finally, Shapiro and Huh (2021) showed that if no credible and verifiable way to disclose quality information exists and no market frictions can be observed, equilibria with partial separation of low-quality sellers can exist as long as buyers are loss averse.

In this paper, we build upon Shapiro and Huh (2021), thereafter SH. The loss-averse preferences, used in SH, have two limitations: they can violate the first-order stochastic dominance and they are sensitive to the choice of the reference point. SH have shown that within their framework, when the seller is a monopolist, a violation of the first-order stochastic dominance is necessary for equilibria with negative information to exist (SH, Proposition 1 and the discussion afterwards). More precisely, there must exist buyers whose loss aversion is so strong they strictly prefer a product with a certain quality of v_L over a lottery where product's quality can either be v_L or v_H, where $v_H > v_L$. These counterintuitive preferences can exist under the loss-aversion framework but do not exist under a rational expected-utility framework.[2] As to the sensitivity to the reference point, SH used a common assumption where the reference point is defined endogenously as the expected consumption utility. However, another natural reference point, zero, can be found. In the SH analysis of the duopoly framework, buyers' utility is positive regardless of the quality they acquire. Thus, if

[2] To see how loss aversion can violate the first-order stochastic dominance, consider the following example. Let $v_L = 1$ and $v_H = 3$, and $\Pr(v = v_L) = 1/2$. Let the reference point be the expected quality, which is 2. That is, if an agent gets a quality below that expected, he experiences a loss. In this example, getting the high-quality product is a gain and the utility is 3. Getting the low-quality product is a loss and the utility is $1 + b(1 - 2)$. Here $b > 0$ is the degree of loss-aversion, 2 is the reference point, and $1 - 2$ is the size of the loss, $v_L - E_v$. Thus, agent's utility is $1/2 \cdot 3 + 1/2 \cdot (1 + b(1 - 2)) = 2 - b / 2$. It is less than v_L when $b > 2$.

different from the SH framework, the reference point is zero then buyers are effectively risk-neutral because they do not experience loss, and no equilibrium with negative information would exist.

Standard expected-utility preferences do not have these limitations: they satisfy the first-order stochastic dominance and do not depend on the reference point. Thus, despite people's choices being able to violate first-order stochastic dominance (Kourouxous and Bauer 2019) and the definition of the reference point used in SH is common and *"... has proven quite popular in applications"* (Masatlioglu and Raymond 2016, p. 2765), a natural question to ask is what happens if we consider the expected-utility framework rather than loss aversion.

To answer this question, we consider a framework where, just like in SH, there are no market frictions and sellers can only use cheap talk messages to communicate quality information. No standard tools that allow high-quality sellers to credibly reveal their quality: no-repeat purchases, no reputation concerns, no certification technology, and no warranties, are available. The only thing that affects buyers' beliefs on the product's quality is a cheap-talk message which means that low-quality sellers can costlessly imitate any communication employed by high-quality sellers. Unlike SH, buyers are risk-averse with the CARA utility function. We show that in this setting, despite limiting high-quality sellers' ability to separate, informative equilibria where low-quality sellers do not claim high quality can exist.

Two factors make it optimal for low-quality sellers to reveal negative information: buyers' risk aversion and competition. We show that both factors (risk aversion and competition) are necessary for equilibrium disclosure of negative information. When the seller is a monopolist or buyers are risk-neutral, no equilibrium with negative information disclosure exists. In a monopoly, the seller's profit is determined entirely by buyers' WTP. When buyers are risk-averse, disclosing negative information has a negative effect on WTP because risk aversion does not violate the first-order stochastic dominance. Therefore, it is suboptimal for low-quality sellers to either separate or partially separate. In a duopoly, the seller's profit depends not only on buyers' WTP but also on the intensity of competition. When buyers are risk-neutral, regardless of the cheap talk messages sent by the duopolists, the Bertrand competition ensues and a seller with lower expected quality is guaranteed to make zero profits. Similarly, neither separation nor partial separation is possible in equilibrium.

Meanwhile, when there are two sellers *and* buyers who are risk-averse, then equilibria with negative information can exist. For this to happen, sufficiently risk-averse buyers must exist but their share should be sufficiently low. To see the intuition, let m_L denote a cheap-talk message "*my quality is low*" and m_H denote a cheap-talk message "*my quality is high*". For the sake of an example, consider the case when seller i sent message m_L, seller j sent message m_H, and buyers believe that seller i has low quality with probability 1 while seller j can have either low or high quality. Because the buyers are risk-averse and thus, their preferences satisfy first-order stochastic dominance, they all have a higher WTP for j's product than for i's product. At the same time, for buyers who are more risk-averse, the difference in WTP between seller j and seller i is smaller. In this setting, the only chance for seller i to make a positive profit is if he can attract the most risk-averse buyers, i.e., those buyers for whom the difference in WTP for the two products is the lowest. For this to happen, seller j must not be interested in competing for those buyers, which, as we show in the paper, happens when there are buyers who are sufficiently risk-averse but their share is small, because otherwise, seller j would find it optimal to serve them as well.

The rest of the paper is organized as follows. In Section 2 we provide a literature review. Section 3 outlines the setup of the model. Subsection 4.1 consider two benchmarks, monopoly and duopoly with risk-neutral buyers, and shows that equilibrium with negative information does not exist. Subsection 4.2 considers the main case of the duopoly with risk-averse buyers and derives conditions when equilibrium with negative information exists. All the proofs are in Appendix A.

II. Literature Review

This study analyzes the motivation of sellers' voluntary information disclosure based on two factors: buyers' risk aversion and the competitive environment. Bauer (1960) was the first to consider that buyers' perceived risk had a strong influence on consumer choice; many marketing studies that followed his work found that perceived risk had a negative effect on buyers' purchase intention (Dowling 1986; Markin 1974; Ross 1975; Stone and Winter 1985; Taylor 1974). According to these studies, incomplete information may increase buyers' perceived risk and decrease their WTP. When lying is prohibited, less-than-full disclosure signals low quality to buyers and the market ends up with voluntary full disclosure (Grossman and Hart 1980; Milgrom 1981; Grossman, 1982).[3] Our paper is related to this literature; however, we relax the assumption of no lying to provide a more practical understanding of the sellers' disclosure of information.

This work also belongs to a series of studies that have shown how sellers may utilize information disclosure to control pressures from the competitive environment (see Guo and Zhao 2009; Board 2009; Gardete 2013; Kim 2012). Unlike earlier papers, we show that when buyers are risk-averse, sellers are incentivized to communicate negative information in purely cheap talk environments with no market friction or certification technology.

Our paper has shown that voluntary disclosure of negative information can occur in one-off or non-repeat purchase situations. When interactions between buyers and the seller are repeated, it is well established that sellers might have incentives to reveal negative information to build their reputations (Farrell 1980; Heal 1976; Riordan 1986; Shapiro 1983; Smallwood and Conlisk 1979; Wilson 1985) or maintain long-term relationships with buyers (Busch and Wilson 1976; Crosby *et al.* 1990; Geyskens *et al.* 1998).

III. Model

There are two risk-neutral sellers. The seller's quality is exogenously given and is pure private information, *i.e.*, it is unobserved by the buyers and by the other seller. There are two quality types, $v \in \{v_L, v_H\}$, where $v_H > v_L$, and the marginal cost is normalized to zero.[4] The probability of the product having low quality is q. Sellers maximize their expected profits

$$U_s(p) = \Pr(sale) \cdot p$$

[3] Information disclosure literature supporting mandatory disclosure has not generally assumed risk aversion (see Hotz and Xiao 2013; Cheong and Kim 2004; Viscusi 1978).

[4] The assumption that marginal cost does not depend on quality is common in the literature (see Shaked and Sutton 1982; or Board, 2009). We impose it because in the case of asymmetric cost information the analysis of even simple Bertrand-like competition framework is extremely complex (Spulber, 1995). For the anlysis of the Cournot competition in the case of asymmetric cost information, see Ryu and Kim (2011).

Sellers can communicate their product's quality to buyers by using cheap talk messages. There are two possible cheap talk messages: m_L and m_H. We interpret these messages as "*my quality is* υ_L" and "*my quality is* υ_H".

Buyers are risk-averse and have a concave Bernoulli utility function, $u(x) = 1 - e^{-\gamma x}$, with constant absolute risk aversion.[5] Let q_μ denote buyer's beliefs that a product has low quality. Then, buyers' expected utility from purchasing this product at price p is

$$U_\gamma(p,\mu) = q_\mu u(\upsilon_L - p) + (1 - q_\mu)u(\upsilon_H - p), \tag{1}$$

where γ is the buyer's degree of absolute risk aversion. While $u(x)$ also depends on γ , we are going to omit it for the notational brevity. Given the CARA assumption, we do not need to specify the initial wealth. Buyers differ in their degree of absolute risk aversion, γ. We assume that buyers' risk aversion γ is distributed with a positive, differentiable log-concave density $\psi(\gamma)$ on support $[0,\Gamma]$. We will use $\psi(\gamma)$ to denote a cumulative distribution function of γ.

The game has three stages. The first stage is the messaging stage, where both sellers simultaneously send cheap talk messages (m_i,m_j) that are publicly observed. The second stage is the pricing stage. Given (m_i,m_j), sellers simultaneously determine prices for their products (p_i,p_j). The third stage is the purchasing stage. Buyers observe the messages and prices of both sellers and choose which product to purchase.

Definition 1. *An equilibrium is a quadruple of sellers' messaging and pricing strategies:* $m_i(\upsilon_i)$, $m_j(\upsilon_j)$, $p_i(m_i, m_j, \upsilon_i)$, $p_j(m_i, m_j, \upsilon_i)$), *buyers' beliefs* ($\mu_i(m_i)$, $\mu_j(m_j)$), *buyers' purchasing strategies, and market demands* ($s_i(\mu_i, \mu_j, p_i, p_j)$, $s_j(\mu_i, \mu_j, p_i, p_j)$) *such that*

i) $m_i(\upsilon_i)$ *maximizes seller i's profit given buyers' beliefs and seller j's strategies:*

$$\max_{m_i \in \mathrm{M}} E_{\upsilon_j}[p_i \cdot s_i(\mu_i, \mu_j, p_i, p_j)]$$

ii) $p_i(m_i, m_j, \upsilon_i)$ *maximizes i's profit given* m_i, m_j, *buyers' beliefs and*

[5] In addition to the functional form used in the paper, there are other representations of CARA utility functions, such as $(1 - e^{-\gamma x})/\gamma$ and $-(e^{-\gamma x})/\gamma$. All these representations are equivalent.

seller j's strategies:

$$\max_{p_i} E_{\upsilon_j}[p_i \cdot s_i(\mu_i, \mu_j, p_i, p_j)]$$

iii) buyers purchasing decisions are optimal and they determine market demand for each firm:
$s_i(\mu_i, \mu_j, p_i, p_j) = Pr(\{U_\gamma(p_i, \mu_i) \geq U_\gamma(p_j, \mu_j)\})$;

iv) if m_i *is sent with a positive probability,* $\mu_i(m_i)$ *is derived from* $m_i(\upsilon_i)$ *following Bayes' rule.*

From the definition above, we assume prices are determined after the message and not jointly. While it is not uncommon for advertising to come with price information, the assumption is appropriate as long as it is quicker to adjust pricing strategy so that sellers can react with their pricing decision to the type of information disclosed by the competitor (Janssen and Teteryatnikova 2016). In our model, messages sent by the sellers determine the intensity of competition. We allow equilibrium prices to reflect the competition's intensity by allowing prices to depend on messages.

Another assumption is that buyers' beliefs are determined by cheap-talk messages only, and in particular, are not affected by prices. In our setup, we intentionally removed the standard mechanisms identified in the literature as a way for high-quality sellers to credibly signal their quality. The reason for this is that limiting the ability of high-quality sellers to communicate their quality places the focus on the incentives of low-quality sellers, who, in our setup, can costlessly imitate any strategy pursued by high-quality sellers if they choose to do so. If any information on a low-quality product is revealed, and therefore, it is driven not by high-quality sellers' ability to separate, as is the case in unraveling or education-as-signaling models, but by the intent of low-quality sellers to not pool with high-quality sellers.[6]

[6] If we extend the current model to the setting where buyers use messages *and* prices to form their beliefs about the quality, then it would greatly expand the set of possible equilibria. Importantly, however, the equilibria that we describe in this paper will remain the equilibria of the extended model. For example, let p_i^* be an on-equilibrium price of the original model given (m_i, m_j, υ_i). In the extended model, we easily generate the same pricing strategy by setting off-equilibrium beliefs so that $\mu(m_i, p_i) = \{\upsilon_L$ with probability $1\}$ for any $p_i \neq p_i^*$.

The empirical literature has explored the circumstances under which either weak or no relationship between prices and *perceived* quality exists. The price-perceived quality relationship is weak in products that are high in experience and credence attributes, which are the types of products that we focus on in the paper. Rao and Monroe (1989) and Lichtenstein and Burton (1989) found that customers are not capable of predicting quality from prices for durable, higher-priced, or non-frequently purchased products. Similarly, Vöckner and Hofmann (2007), in a meta-study that covered 23 papers on the price-perceived quality relationship, have shown that durable goods as well as services, which are generally low in search attributes and high in experience and credence attributes, have much weaker price-perceived quality relationships than fast-moving consumer goods.

Finally, we ignore participation constraints and assume that buyers always purchase a product from one of the firms. In particular, $s_i(\mu_i, \mu_j, p_i, p_j) + s_j(\mu_i, \mu_j, p_i, p_j) = 1$ so that sellers always compete directly with each other. This assumption is satisfied when buyers' valuation is sufficiently high so that participation constraints are not binding. Appendix B discusses how our analysis changes when participation constraints are binding.

IV. Equilibria with Negative Information

In this paper, we are interested in equilibria where the low-quality type separates with positive probability. Consider messaging strategy $m^*(\upsilon) = (\lambda, 1)$. According to this strategy, a low-quality seller mixes between messages m_L and m_H with probabilities $\lambda > 0$ and $1 - \lambda$; a high-quality seller sends a message m_H with probability 1. If the seller plays according to $m^*(\upsilon)$ and the buyers receive a message m_L, they know that the seller's quality is υ_L with probability 1. We will refer interchangeably to the equilibria where the sellers play $m^*(\upsilon)$ as *equilibria with partial separation* or *equilibria with negative information*.

A. Two Benchmarks: Monopoly and Duopoly with Risk-Neutral Buyers

First, we consider two benchmarks. The first benchmark is the monopoly setting with risk-averse buyers. The second benchmark is the duopoly setting with risk-neutral buyers. We will show that in either benchmark, the equilibrium with partial separation of the low-quality type does not exist. It means that within the expected utility framework, risk aversion *and* duopoly are necessary for equilibrium with disclosure of negative information to exist.[7]

a) Monopoly Risk-Averse Buyers

For the monopoly benchmark, Definition 1 needs to be adjusted to the case of the monopolistic seller. The main adjustment is in part c), where we require that buyers purchase the product from the monopolist if and only if their expected utility from purchasing is non-negative.

Proposition 1. *In the framework with the monopolistic seller, equilibria with negative information do not exist.*

In the monopoly setting, the seller's profit is determined by buyers' WTP. If the low-quality type separates then buyers' willingness to pay is equal to v_L. If the low-quality type pools with the high-quality type then buyers believe that the seller's quality is randomly distributed on support $\{v_L,v_H\}$. When buyers are risk-averse, their WTP in the latter case is higher than v_L. Therefore, it is never optimal for the low-quality seller to separate.

If one considers behavioral risk preferences, such as loss aversion, then equilibrium with the partial separation of the lowest-quality type can exist. However, as shown in Shapiro and Huh (2021), a necessary condition for equilibrium existence is that the most loss-averse buyer strictly prefers a product with the certain quality v_L over the product whose quality can be either v_L or v_H. Within the standard expected-utility framework considered in this paper, such counterintuitive preferences are impossible so the equilibrium does not exist.

[7] The main results of this subsection—equilibria with negative information do not exist under the two benchmarks—does not mean that there are no other equilibria. For example, it is immediate to verify that for both benchmarks there exists a babbling equilibrium where buyers ignore cheap-talk messages so that their posterior beliefs are equal to their prior beliefs. In the case of monopoly, buyers purchase the product if their utility is greater or equal than zero, $U_\gamma(\mu,p) \geq 0$, where μ is the prior and p is the seller's price. The monopolistic seller sets the price that maximizes its profit: $p \cdot \Pr(\gamma \mid U_\gamma(\mu,p) \geq 0)$. In the case of duopoly, buyers have identical beliefs about products of both sellers and, therefore, the babbling equilibrium is the Bertrand competition where both sellers set prices equal to zero.

b) Duopoly. Risk-Neutral Buyers

Proposition 2. *If buyers are risk-neutral, the equilibria with negative information do not exist.*

In the case of duopoly, the effect of revealing negative information on profit is different. Now, the seller's profit depends not only on buyers' WTP but also on the intensity of competition. When buyers are risk-neutral, then for any two messages (m_i,m_j), the resulting competition is essentially the Bertrand competition, and all buyers purchase the product from the seller who offers the highest expected consumer surplus. A seller with lower expected quality is guaranteed to make zero profit in equilibrium. Therefore, the duopoly setting with risk-neutral buyers does not add any benefits to negative information disclosure. The equilibrium with partial separation does not exist.

B. Duopoly with Risk-Averse Buyers

We now show that when buyers are risk-averse and there are two sellers, there exist equilibria with the disclosure of negative information. In this setting, revealing negative information can introduce product differentiation and soften the competition in the presence of risk-averse buyers. As defined earlier, let $m^*(\upsilon)$ be a messaging strategy where a low-quality seller mixes between messages m_L and m_H with probabilities λ and $1-\lambda$, and a high-quality seller sends a message m_H with probability 1. We will determine conditions when $m^*(\upsilon)$ is a messaging strategy in a symmetric equilibrium.

By backward induction we first look at the purchasing stage. If both sellers send the same messages (m_L,m_L) or (m_H,m_H) , then from the buyers' point of view, the two sellers are identical. Therefore, buyers will purchase the product with the lowest price, which results in Bertrand competition with both sellers charging $p_i = p_j = 0$.

Consider now the purchasing decision given messages (m_L,m_H), and prices (p_L,p_H). Given sellers' strategy, the m_L-product has a certain quality of υ_L. The m_H-product has low quality with a probability $q_H = Pr(\upsilon = \upsilon_L | m_H)$, where

$$q_H = \frac{(1-\lambda)q}{(1-\lambda)q + (1-q)}. \tag{2}$$

A buyer is indifferent between m_L- and m_H-products if

$$u(v_L - p_L) = q_H u(v_L - p_H) + (1 - q_H)u(v_H - p_H), \tag{3}$$

or, equivalently, if γ^0 is such that

$$e^{-\gamma^0(p_H - p_L)} - q_H - (1 - q_H)e^{-\gamma^0(v_H - v_L)} = 0. \tag{4}$$

Proposition 3 shows that buyers with a degree of absolute risk aversion below γ^0 will purchase from the m_H-seller, and buyers with γ above γ^0 will purchase from the m_L-seller.

Proposition 3. *Given messages* (m_L, m_H). *the indifference condition* $e^{-\gamma^0(p_H - p_L)} = q_H + (1 - q_H)e^{-\gamma^0(v_H - v_L)}$, *has at most one solution* $\gamma^0 > 0$. *When the solution exists, all buyers with* $\gamma > \gamma^0$ *prefer an* m_L *-product, while all buyers with* $\gamma > \gamma^0$ *prefer an* m_H *-product.*

One can see that, different from the risk-neutral case, the competition intensity varies depending on the message profile. When the two messages are the same, (m_L, m_L) or (m_H, m_H), the competition is an intense Bertrand competition and both sellers earn zero profit. However, when the two messages differ, (m_L, m_H), the competition intensity is weaker as the products become differentiated. Buyers with a lower degree of risk aversion prefer a product with higher expected quality, while buyers with a higher degree of risk aversion prefer a product with a certain quality even if the quality is low. The effect of product differentiation can be observed from the indifference condition (4). Small changes in prices will have a small effect on γ^0 and consequently on the sellers' market shares, which lowers sellers' incentives to undercut each other, thereby softening competition and allowing sellers' to earn positive profit.

From Proposition 3, it follows that demand for the m_L-product is $(1 - \Psi(\gamma^0(p_L, p_H))$, and demand for the m_H-product is $\Psi(\gamma^0(p_L, p_H))$. The m_L-seller chooses the price p_L to maximize $\max_{pL} (1 - \Psi(\gamma^0(p_L, p_H)) \cdot p_L$, and the m_H -seller chooses the price p_H to maximize $\max_{pH} \Psi(\gamma^0(p_L, p_H)) \cdot p_H$. The corresponding first-order conditions are

$$-\psi(\gamma^0)\frac{\partial \gamma^0(p_L, p_H)}{\partial p_L}\, p_L + (1 - \Psi(\gamma^0)) = 0, \tag{5}$$

and

$$\psi(\gamma^0)\frac{\partial \gamma^0(p_L, p_H)}{\partial p_H}\, p_H + \Psi(\gamma^0) = 0. \tag{6}$$

Finally, at the messaging stage, a low-quality seller should be indifferent between m_L and m_H:

$$\lambda q \pi_{m_L,m_L} + (1 - \lambda q)\pi_{m_L,m_H} = \lambda q \pi_{m_H,m_L} + (1 - \lambda q)\pi_{m_H,m_H}. \tag{7}$$

Here, the LHS is the expected profit of the low-quality seller from sending a message m_L and the RHS is the expected profit from sending a message m_H, λq is the probability that the competitor sends a message m_L, and $(1 - \lambda q)$ is the probability that the competitor sends a message m_H, and π_{m_i,m_j} is the profit of seller i given message profile (m_i,m_j). As established earlier, $\pi_{m_L,m_L} = \pi_{m_H,m_H} = 0$.

Combining (4), (5), (6), and (7), an equilibrium is determined by the following:

$$\begin{cases} e^{-\gamma^0(p_H - p_L)} = q_H + (1 - q_H)e^{-\gamma^0(v_H - v_L)}. \\ -\psi(\gamma^0)\dfrac{\partial \gamma^0(p_L, p_H)}{\partial p_L} p_L + (1 - \Psi(\gamma^0)) = 0 \\ -\psi(\gamma^0)\dfrac{\partial \gamma^0(p_L, p_H)}{\partial p_L} p_H + \Psi(\gamma^0) = 0 \\ (1 - \lambda q)(1 - \psi(\gamma^0))p_L = \lambda q \Psi(\gamma^0)p_H. \end{cases} \tag{8}$$

Proposition 4. *The equilibrium in which the low-quality type separates does not exist if*

i) $\Psi(\gamma)$ is a uniform distribution or

ii) $\Psi(\gamma)$ is a convex function.

The equilibrium where the lowest-quality type separates exists if

iii) $\Psi(\gamma)$ has infinite support.

Furthermore,

iv) for any concave $\Psi(\gamma)$, there exists $\alpha^0 > 0$ such that for any $\alpha \in (0,\alpha^0)$, if risk aversion is distributed with cdf $\Psi(\alpha\gamma)$, the equilibrium where the lowest-quality type separates exists.

Intuitively, consider the pricing subgame after messages (m_L,m_H). In terms of quality, the m_H-product is superior to the m_L-product. The m_L-product is guaranteed to be of low quality, while the m_H-product can be of either low or high quality. Regardless of γ, all risk-averse buyers have higher WTP for the m_H-product. However, for those who are more risk-averse, the difference in WTP between the m_H- and m_L-products is smaller. Thus, the only way the m_L-seller can obtain a positive share of the market is if it can attract risk-averse buyers with high γ. The m_H-seller's willingness to compete for those buyers depends on two factors: a) how high γ can get and b) how large the share of buyers with high γ is. If Γ is low or if there are too many buyers with high γ, then it is optimal for the m_H-seller to simply outprice the m_L-seller and serve the whole market. In the first case, when Γ is low, even the most risk-averse buyers are not overly concerned with quality uncertainty. The m_H-seller does not need to discount his product much to attract those buyers. In the second case, when there is a sufficient number of buyers with high risk aversion, it is suboptimal for m_H to choose not to serve them. Thus, only when a sufficient number of risk-averse buyers *and* their share is sufficiently small exist is it possible to have an equilibrium where v_L separates.

Proposition 4 captures the notion of having "*sufficiently risk-averse buyers and their share is sufficiently low*" using convexity and support of $\Psi(\gamma)$. For any $\Psi(\gamma)$ with infinite support, the equilibrium with the lowest-type separation exists. First, there are sufficiently risk-averse buyers. Second, there exists $\tilde{\gamma}$ high enough that $\Psi(\gamma)$ can be made arbitrarily small for any $\gamma > \tilde{\gamma}$. The marginal benefit of decreasing p_H to serve buyers with $\gamma > \tilde{\gamma}$ will then be too small due to the low increase in the market share. In equilibrium, m_L- and m_H-sellers will split the market.

For uniform or convex cdfs, regardless of how high Γ is, $\Psi(\gamma)$ never approaches 0 for high values of γ. A marginal decrease in p_H to attract buyers with high risk aversion is always optimal, and seller m_L will be priced out of the market. Meanwhile, with concave distributions, most buyers have a low degree of risk aversion. However, concavity alone is not enough to guarantee the existence of equilibrium. $\Psi(\gamma)$ must also be sufficiently small when γ is close to Γ. One way to do this is to stretch $\Psi(\gamma)$ to a larger support, and Proposition 4 provides one way for this to be achieved.

V. Conclusion

Although we often observe sellers' voluntary disclosure of negative information in many markets, existing literature cannot fully explain the rationale behind the sellers' counterintuitive behavior. In this paper, we analyzed low-quality sellers' motivation to separate themselves from high-quality sellers by focusing on the risk attitude of buyers in the market. Using a duopoly model with risk-averse buyers, we show that buyers' risk aversion and competitive pressure can make it optimal for low-quality sellers to voluntarily disclose negative information on their products or services. More specifically, when there are sufficiently risk-averse buyers but their share is low, low-quality sellers can carve out a niche wherein they reveal their low quality and serve most risk-averse buyers.

This paper suggests that sellers should consider buyers' risk aversion in markets under information asymmetry when determining strategies for disclosing relevant product information. Despite the conventional wisdom that sellers should conceal negative information, it may be profitable for sellers to voluntarily reveal negative aspects of their offerings to risk-averse buyers.

In the example mentioned above, Avis chose to voluntarily disclose that their services were inferior to those of their competitor. This strategy was successful and resulted in increased sales and profit. From this outcome, we can infer that rental car customers had high uncertainty as to the quality of services provided by lesser-known rental car companies. Therefore, Avis's strategy distinguished it from Hertz, the leading company at the time, and communicated its quality level to potential customers. This approach appealed to risk-averse customers in particular and resulted in increased revenue and profit. At the same time, there may have been a relatively low number of highly risk-averse customers, which prevented Hertz from retaliating to Avis's marketing strategy.

The results of our paper also suggest some important implications for policymakers in terms of solving adverse selection issues in markets with information asymmetry. Because buyers' risk aversion can encourage sellers to voluntarily disclose information, it may be an efficient strategy to educate buyers to be more aware of possible sellers' fraud in advance. As a result, they may perceive more risk in markets with high levels of information asymmetry and higher chances of seller fraud. By increasing buyer risk awareness, policymakers can encourage sellers to voluntarily reveal negative aspects of their offerings and solve potential adverse selection issues under information asymmetry.

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Appendix A: Proofs

Proof of Proposition 1: Consider a messaging strategy $m^{*}(v)$, as defined above, where type v_L separates with positive probability with message m_L being the separating message: $Pr(v = v_L | m_L) = 1$. Conditional on m_L, no uncertainty associated with the product and buyers' willingness to pay exist regardless of the degree of their risk aversion, is v_L. The optimal monopolist price is v_L, and so is the seller's profit.

An alternative of imitating high-quality sellers is more profitable. Let $q_H = Pr(v = v_L | m_H)$ be beliefs that seller's quality is low conditional on message m_H. For any beliefs, $(q_H, 1 - q_H)$, such that $q_H < 1$, and any degree of risk aversion, the buyer's expected utility, $q_H u(v_L - p) + (1 - q_H) u(v_H - p)$, is strictly positive when $p = v_L$. Thus, there exists sufficiently small $\varepsilon > 0$ such that all buyers will purchase at price $p' = v_L + \varepsilon$, and the low-quality seller would earn a higher profit of $v_L + \varepsilon$. Therefore, separation for the lowest-quality type is strictly less profitable.

Proof of Proposition 2: Consider the pricing subgame that follows messages (m_i, m_j). Denote buyers' beliefs on the expected quality of the two sellers as v_i and v_j and without loss of generality assume that $v_i \geq v_j$. Thus, it is trivial to show that in the equilibrium of this subgame seller j charges price $p_j = 0$ and seller i charges price $p_i = v_i - v_j \geq 0$ so that $\pi_j = 0$ and $\pi_i = v_i - v_j \geq 0$.

Assume that there exists an equilibrium with negative information. In this equilibrium, a low-quality seller, say seller j, sends a message m_L with a positive probability and buyers' beliefs are such that $\Pr(v_j = v_L | m_j = m_L) = 1$. Then, conditional on sending a message m_L , seller j earns zero profit because for any on-equilibrium message m_i sent by seller i, buyers' beliefs on the quality of seller i are weakly higher than v_L. By the argument above, seller j will earn zero profit in any pricing subgame that follows a message m_L .

It leads to a contradiction. On the one hand, because the message m_L is an on-equilibrium message it means that seller j must earn zero expected profit after any on-equilibrium message. Otherwise, it is suboptimal to send m_L . Therefore, in any equilibrium where seller j discloses negative information, he earns zero expected profit. On the other hand, seller j must earn a positive expected profit in this equilibrium. Indeed, there must exist an on-equilibrium message $m_j \neq m_L$ such that $E(v_j | m_j') > Ev$, where Ev is prior expected quality and $E(v_j | m_j')$ are buyers' beliefs conditional on m_j'.[8] At the same time, for any messaging strategy of seller i , there must exist an on-equilibrium message m_i' such that $E(v_i | m_i') \leq Ev$. Then in a subgame that follows (m_i', m_j') seller j will earn a positive profit. Because both messages are on-equilibrium, the message profile (m_i', m_j') will happen with positive probability and thus, the expected profit of seller j is positive. Contradiction.

Proof of Proposition 3: Let y denote $e^{-\gamma^0}$ so that (4) is

$$y^{p_H - p_L} - q_H - (1 - q_H)y^{v_H - v_L} = 0. \tag{9}$$

Because γ^0 varies between 0 and $+\infty$, variable y varies between 1 and 0. There is one solution $y = 1$, which corresponds to $\gamma = 0$. To prove the proposition, we need to show that on interval $0 \leq y < 1$ there is at most one solution. Given the root $y = 1$, it is equivalent to showing that there are at most two solutions of (9) on $0 \leq y \leq 1$.

Assume not. If function (9) has three or more roots on [0,1], then, its derivative should have two or more roots on [0,1]. Taking the derivative of the LHS of (9) with respect to y and setting it equal to zero we get the following:

$$(p_H - p_L)y^{p_H - p_L - 1} - (1 - q_H)(v_H - v_L)y^{v_H - v_L - 1} = 0,$$

so that

$$1 - (1 - q_H)\frac{v_H - v_L}{p_H - p_L}\, y^{(v_H - v_L) - (p_H - p_L)} = 0.$$

The equation above has at most one solution on interval [0,1] and, therefore, equation (9) has at most two solutions on [0,1]. Because one solution is $y = 1$, it implies that there is at most one solution when $0 < y < 1$.

To prove the second part of the proposition, we observe that buyers prefer the m_L-product if

[8] If there are exactly two messages then $m_j' = m_H$. However, the argument does not depend on the requirement that there are only two possible cheap talk messages.

$$1-e^{-\gamma(v_L-p_L)} \geq q_H(1-e^{-\gamma(v_L-p_H)})+(1-q_H)(1-e^{-\gamma(v_H-p_H)}),$$

which is equivalent to

$$e^{-\gamma(p_H-p_L)} \leq q_H+(1-q_H)e^{-\gamma(v_H-v_L)}. \tag{10}$$

When $\gamma=+\infty$, then (10) is satisfied as the LHS is zero, and the RHS is positive. In other words, extremely risk-averse buyers will always purchase the m_L-product. Thus, if $\gamma^0>0$ is the solution to (4), then all types with $\gamma>\gamma^0$ will purchase the m_L-product. Customers with $\gamma<\gamma^0$ will purchase the m_H-product, because as established earlier, the indifference condition holds with equality when $\gamma=\gamma^0$ and $\gamma=0$. Were it the case that customers with $0<\gamma<\gamma^0$ prefer the m_L-product, it would mean that derivatives of the LHS and the RHS of (10) are equal to each other twice: once at γ^0 and once at some point in $(0,\gamma^0)$. However, it contradicts the earlier established fact that the derivatives of LHS and RHS can be equal to each other for at most one value of γ.

Proof of Proposition 4: The proof of the proposition consists of two parts. In the first part, we reduce the equilibrium system (8) to one equation with one unknown, γ^0. In the second part, we analyze that equation and develop sufficient conditions on $\Psi(\gamma)$ stated in Proposition 4.

We begin the first part by noting that to prove the existence, we can ignore the last equation in (8) and variable λ. We will refer to system (8) without the last equation as the *reduced system*. Once we solve the reduced system for (γ^0,p_L,p_H), we can recover the values of q and λ from the fourth equation. This is formally established in the next Lemma.

Lemma 1. *Fix $q_H\in(0,1)$ Let $(\bar{\gamma}^0, \bar{p}_L, \bar{p}_H)$ be a solution to the reduced system such that $\bar{\gamma}^0\in(0,\Gamma)$, and $\bar{p}_L, \bar{p}_H>0$. Then there exists $q\in(0,1)$ and $\lambda\in(0,1)$ such that q_H is defined by (2) and $(\bar{\gamma}^0, \bar{p}_L, \bar{p}_H, \lambda)$ is a solution to the equilibrium system (8).*

Proof. Plug values $(\bar{\gamma}^0, \bar{p}_L, \bar{p}_H)$ into the last equation of (8) to recover λq. Notice that for any $(\bar{\gamma}^0, \bar{p}_L, \bar{p}_H)$ that satisfy conditions of Lemma 1 one can find λq such that it is between 0 and 1 and the last equation of (8) is satisfied. From λq and q_H one can then uniquely recover q and λ: $q=1-(1-q_H)(1-\lambda q)$ and $\lambda=(\lambda q)/q$, where we use (2) for the first equation.

To complete the proof one needs to verify that $q\in(q_H, 1)$ and $\lambda\in(0,$

1). The requirement that $q > q_H$ comes from the fact that the share of low-quality sellers who announce high quality, q_H, cannot be higher than the total share of low-quality sellers, q. It also cannot be equal to q because we focus on equilibria where low-quality sellers reveal negative information with positive probability.

First, trivially, $q = 1 - (1 - q_H)(1 - \lambda q) < 1$. As for $q_H < q$:

$$q_H < q = 1 - (1 - q_H)(1 - \lambda q) = q_H + \lambda q - q_H\lambda q.$$

Finally, $\lambda = (\lambda q)/q$ is positive because both λq and q are positive. As for $\lambda < 1$ it is equivalent to $\lambda q < q$, which is true:

$$\lambda q < q = 1 - (1 - q_H)(1 - \lambda q) = q_H + \lambda q - q_H\lambda q = \lambda q + q_H(1 - \lambda q).$$

This completes the proof.

The reduced system has three equations and three unknowns (γ^0, p_L, p_H), and it treats q_H as a given parameter. The three equations of the reduced system are the indifference condition

$$e^{-\gamma(p_H - p_L)} = q_H + (1 - q_H)e^{-\gamma^0(v_H - v_L)}, \tag{11}$$

and two FOCs that determine prices are as follows:

$$\begin{cases} -\psi(\gamma^0)\dfrac{\partial\gamma^0(p_L, p_H)}{\partial p_L}\, p_L + (1 - \Psi(\gamma^0)) = 0 \\ -\psi(\gamma^0)\dfrac{\partial\gamma^0(p_L, p_H)}{\partial p_L}\, p_H + \Psi(\gamma^0) = 0 \end{cases}. \tag{12}$$

Take the second equation of (12) and subtract from it the first equation of (12). We get

$$\frac{\Psi(\gamma^0)}{\psi(\gamma^0)} - \frac{1 - \Psi(\gamma^0)}{\psi(\gamma^0)} = \frac{\partial\gamma^0(p_L, p_H)}{\partial p_L}(p_H - p_L). \tag{13}$$

It will be convenient to denote $\dfrac{\Psi(\gamma^0)}{\psi(\gamma^0)} - \dfrac{1 - \Psi(\gamma^0)}{\psi(\gamma^0)}$ as $A(\gamma^0)$. From (11) we get that

$$\frac{\partial \gamma^0}{\partial p_L} = -\frac{\partial \gamma^0}{\partial p_L} = -\frac{\gamma^0 e^{-\gamma^0(p_H, p_L)}}{-(p_H - p_L)e^{-\gamma^0(p_H, p_L)} - (1 - q_H)(v_H - v_L)e^{-\gamma^0(v_H, v_L)}}$$
$$= \frac{\gamma^0 e^{-\gamma^0(p_H, p_L)}}{(p_H - p_L)e^{-\gamma^0(p_H, p_L)} - (1 - q_H)(v_H - v_L)e^{-\gamma^0(v_H, v_L)}}.$$

Let $\Delta p = p_H - p_L$ and $\Delta v = v_H - v_L$. Thus the reduced system becomes a system of two equations and two unknowns:

$$\begin{cases} e^{-\gamma^0 \Delta p} = q_H + (1 - q_H)e^{-\gamma^0 \Delta v} \\ A(\gamma^0) = \dfrac{\gamma^0 e^{-\gamma^0 \Delta p}}{\Delta p e^{-\gamma^0 \Delta p} - (1 - q_H)\Delta v e^{-\gamma^0 \Delta v}} \Delta p \end{cases}. \tag{14}$$

We can now eliminate Δp from (14) and reduce it to one equation. First, we re-write the second equation of (14) as

$$A(\gamma^0)\Delta p e^{-\gamma^0 \Delta p} - A(\gamma^0)(1 - q_H)\Delta v e^{-\gamma^0 \Delta v} = \gamma^0 \Delta p e^{-\gamma^0 \Delta p}$$
$$A(\gamma^0)\Delta p - A(\gamma^0)(1 - q_H)\Delta v e^{-\gamma^0 \Delta v} e^{\gamma^0 \Delta p} = \gamma^0 \Delta p$$
$$A(\gamma^0)(1 - q_H)\Delta v e^{-\gamma^0 \Delta v} \frac{e^{\gamma^0 \Delta p}}{\Delta p} = A(\gamma^0) - \gamma^0$$
$$\frac{\gamma^0 A(\gamma^0)(1 - q_H)\Delta v e^{-\gamma^0 \Delta v}}{A(\gamma^0) - \gamma^0} = \gamma^0 \Delta p e^{-\gamma^0 \Delta p}. \tag{15}$$

Second, from the indifference condition, we get that

$$\gamma^0 \Delta p = -\ln(q_H + (1 - q_H)e^{-\gamma^0 \Delta v}),$$

and then

Plugging it into (15) we get that the equilibrium value of γ^0 is determined by

$$-\frac{\gamma^0 A(\gamma^0)(1 - q_H)\Delta v e^{-\gamma^0 \Delta v}}{A(\gamma^0) - \gamma^0} = (q_H + (1 - q_H)e^{-\gamma^0 \Delta v}) \ln(q_H + (1 - q_H)e^{-\gamma^0 \Delta v}). \tag{16}$$

This completes the first part of the proof.

In the second part of the proof, we analyze equation (16) and derive conditions that determine whether there exists $\gamma^0 \in (0, \Gamma)$ which is a solution to (16). For the brevity of notations, we will refer to the RHS and LHS of (16) as RHS and LHS without referring to the equation number.

First, we establish that in equilibrium $\Delta p > 0$. Indeed, consider the indifference condition

$$e^{-\gamma^0(p_H - p_L)} = q_H + (1 - q_H)e^{-\gamma^0(\upsilon_H - \upsilon_L)},$$

and take the natural logarithm of both sides:

$$-\gamma^0(p_H - p_L) = \ln(q_H + (1 - q_H)e^{-\gamma^0(\upsilon_H - \upsilon_L)}).$$

The expression inside the logarithm is less than one. Then $\ln(q_H + (1 - q_H)e^{-\gamma^0(\upsilon_H - \upsilon_L)}) < 0$, which implies that $\Delta p > 0$.

Next, we establish that in equilibrium $A(\gamma^0) > \gamma^0$. Because $\Delta p > 0$, it follows from (13) that $\Psi(\gamma^0)/\psi(\gamma^0) > 1 - \Psi(\gamma^0)/\psi(\gamma^0)$ and, therefore, in equilibrium $A(\gamma^0) > 0$. Furthermore, because $\Delta p > 0$, the LHS of (15) must be positive. In equilibrium, $A(\gamma^0) > 0$, which implies that $A(\gamma^0) > \gamma^0$. If such γ^0 does not exist, then (15) cannot be satisfied and no equilibrium with the disclosure of negative information can exist.

Finally, we establish that if $\gamma^0 \in (0, \Gamma)$ such that it is a solution to (16), then the corresponding p_L and p_H are strictly positive. We have also established that $A(\gamma^0) > 0$ and $\Delta p > 0$. From (13), it follows that $\partial\gamma^0(p_L, p_H)/\partial p_L > 0$. From (12), it follows that p_L and p_H are strictly positive. Therefore, once we find $\gamma^0 \in (0, \Gamma)$ that solves (16), we can find positive p_L and p_H that solve the reduced the system. We can then apply Lemma 1 to solve for the remaining parameters.

Proof of i and ii: As we have established earlier, if $A(\gamma) < \gamma$ for every γ, then the solution to (15) and (16) does not exist. We will show that this is the case for uniform and convex distributions. Let the support of $\Psi(\gamma)$ be $[0, \Gamma]$. It is finite for uniform and convex distributions. Inequality $A(\gamma) < \gamma$ is equivalent to $2\Psi(\gamma) - 1 < \gamma\psi(\gamma)$. We will write it as $\Psi(\gamma) - 1 < \gamma\psi(\gamma) - \Psi(\gamma)$. Function $\Psi(\gamma)$ is a weakly convex function such that $\Psi(0) = 0$. By a standard property of convex functions $\gamma\psi(\gamma) \geq \Psi(\gamma) - \Psi(0)$ and because $\Psi(0) = 0$, we have $\gamma\psi(\gamma) \geq \Psi(\gamma)$. Thus, in the inequality $\Psi(\gamma) - 1 < \gamma\psi(\gamma) - \Psi(\gamma)$, the left-hand side is negative and the right-hand side is non-negative, which means it is satisfied for any $\gamma \in [0, \Gamma]$. When $\gamma = \Gamma$ and $\Psi(\gamma)$ is linear, then $A(\Gamma) = \Gamma$, and in all other cases, $A(\Gamma) < \Gamma$. Thus, for the case of convex and uniform distribution functions no equilibrium can be found where both firms split the market.

Proof of iii: The RHS is a continuous function of γ. It is negative for any $\gamma > 0$. When $\gamma = 0$, it is equal to zero. When $\gamma \to \infty$, its limit is equal to $q_H \cdot \ln(q_H) < 0$.

The LHS is discontinuous when $A(\gamma) = \gamma$. Let $\hat{\gamma}$ denote the largest root, such that $A(\gamma) = \gamma$. We can show that it exists. First, $A(0) = -1/\psi(0) < 0$. Second, $\lim_{\gamma \to \infty}\gamma\psi(\gamma) = 0$. If the limit is positive, say $z > 0$, it means that for all sufficiently large γ, say for all $\gamma > \Gamma^0$, it has to be the case that $\psi(\gamma) > 1/2\ z/\gamma$. However,

$$\int_{\Gamma^0}^{\infty} \psi(s)ds > -\int_{\Gamma^0}^{\infty} -d\gamma = \infty,$$

which is a contradiction because it has to be less than or equal to 1. Third,

$$\lim_{\gamma\to\infty}(A(\gamma)-\gamma) = \lim_{\gamma\to\infty}\frac{2\Psi(\gamma)-1-\gamma\psi(\gamma)}{\psi(\gamma)} = \frac{1}{0} = \infty.$$

Given that $A(\gamma) - \gamma$ is continuous, we can conclude that it has roots and that a largest root exists. In other words, there exists $\hat{\gamma}$, such that $A(\hat{\gamma}) = \hat{\gamma}$ and $A(\gamma) > \gamma$ for every $\gamma > \hat{\gamma}$. Therefore, the LHS is a continuous function for any $\gamma > \hat{\gamma}$.

We can now prove the existence of an equilibrium. Because $\hat{\gamma}$ is the largest root, it means that for any $\gamma > \hat{\gamma}$, it must be the case that $A(\gamma) > \gamma$, and in a sufficiently small right neighborhood of $\hat{\gamma}$ fraction $A(\gamma)/A(\gamma) - \gamma)$ is close to plus infinity. The LHS is close to $-\infty$ and is therefore less than the RHS. When γ is close to infinity, the LHS gets arbitrarily close to zero because all terms of the LHS, including $A(\gamma)/\ (A(\gamma) - \gamma)$, are bounded and the term $e^{-\gamma\Delta\upsilon}$ converges to zero. That $A(\gamma)/\ (A(\gamma) - \gamma)$ is bounded follows from

$$\lim_{\gamma\to\infty}\frac{A(\gamma)}{A(\gamma)-\gamma} = \frac{2\Psi(\gamma)-1}{2\Psi(\gamma)-1-\gamma\psi(\gamma)} = 1.$$

Therefore, for sufficiently large γ, the LHS of (16) is less than the RHS. By continuity, the solution to (16) exists.

Proof of iv: Let support of $\Psi(\gamma)$ be $[0, \Gamma]$ where $\Gamma < \infty$. Then $A(\Gamma) > \Gamma$.

Indeed, $A(\Gamma) > \Gamma$ is equivalent to $1 > \Gamma\psi(\Gamma)$. Assume that it is not satisfied so that $\psi(\Gamma) > 1/\Gamma$. Then, because $\Psi(\gamma)$ is strictly decreasing we have

$$1 = \int_0^\Gamma \psi(s)ds > \int_0^\Gamma \frac{1}{\Gamma}ds = 1,$$

which is a contradiction. We can show that $A(\gamma) < \gamma$ when γ is sufficiently close to zero, or more precisely for any γ such that $\Psi(\gamma) < 1/2$. Thus, there exists γ such that $A(\gamma) = \gamma$ and let $\hat{\gamma}$ be the largest such γ. The LHS of (16) is continuous when $\gamma \in (\hat{\gamma},\Gamma]$.

As in case iii), one could try to use continuity to establish that the solution to (16) exists. However, it might not work with the original distribution because unless Γ is sufficiently large, and the LHS will not be close enough to zero to guarantee that the solution exists. Instead, consider a cdf function Ψ_α defined as $\Psi(\alpha\gamma)$. It is a concave function with support $[0,\Gamma,\alpha]$. Now, the largest value of γ is Γ/α. By taking α sufficiently small, we can make the support $[0,\Gamma,\alpha]$ large enough so that $\gamma e^{-\gamma\Delta\upsilon}$ can be made sufficiently close to zero within the support.

Meanwhile, term $A(\gamma)/(A(\gamma) - \gamma)$ will not change. Let $A_\alpha(\gamma)$ be defined similarly to $A(\gamma)$ but with a cdf Ψ_α. Then for any $\gamma \in (0,\Gamma]$,

$$\frac{A_\alpha(\gamma / \alpha)}{A_\alpha(\gamma / \alpha) - \gamma / \alpha} = \frac{A(\gamma)}{A(\gamma) - \gamma}.$$

Indeed,

$$\frac{A_\alpha(\gamma / \alpha)}{A_\alpha(\gamma / \alpha) - \gamma / \alpha} = \frac{2\Psi_\alpha(\gamma / \alpha) - 1}{2\Psi_\alpha(\gamma / \alpha) - 1 - (\gamma / \alpha)\psi_\alpha \gamma / \alpha)}$$
$$= \frac{2\Psi(\gamma) - 1}{2\Psi(\gamma) - 1 - (\gamma / \alpha)\alpha\psi(\gamma)} = \frac{A(\gamma)}{A(\gamma) - \gamma}.$$

Thus, when α is sufficiently small, we can apply the reasoning of case iii) to function $\Psi_\alpha(\gamma)$ to show that a solution exists.

Appendix B: Participation Constraints

In the main part of the paper, we assumed that participation constraints are not binding. In this Appendix, we discuss what happens if we consider participation constraints. Everywhere throughout the paper, we focus on equilibria where low-quality sellers mix between m_L and m_H with probabilities λ and $1-\lambda$, and a high-quality seller sends a message m_H with probability 1.

With participation constraints market demands become

$$s_i(\mu_i, \mu_j, p_i, p_j) = \Pr(U_\gamma(p_i, \mu_i) \geq \max\{U_\gamma(p_j, \mu_j), 0\}). \tag{17}$$

When sellers send the same messages they set prices equal to zero and thus, participating constraints are not binding. Consider now an (m_L,m_H)-subgame and let sellers' prices be (p_L,p_H). If $\upsilon_L < p_L$, then all customers, regardless of their γ, have negative utility from the m_L-product and no one will purchase the product from the m_L-firm. This case is impossible in equilibrium because the m_L-firm then earns zero profits regardless of the message of the competitor. If $\upsilon_L \geq p_L$, then all customers have non-negative utility from the m_L-product. In this case, participation constraints are non-binding and all buyers will purchase the product from either m_L- or m_H-firm.[9] Firms' market shares are determined according to the indifference condition (3) and Proposition 3.

Therefore, equilibria with negative information are still characterized by equilibrium system (8) with an additional constraint that $p_L^* \leq \upsilon_L$. Specifically, let (p_L^*, p_H^*, λ^*) be a solution of (8). If $p_L^* > \upsilon_L$, then the participation constraints are violated and no equilibrium with negative information exists. If $p_L^* \leq \upsilon_L$, then the participation constraints are satisfied and (p_L^*, p_H^*, λ^*) is an equilibrium with negative information.

Following (4), the risk aversion of the indifferent buyer is determined by the difference in quality valuations, $\upsilon_H - \upsilon_L$ and not by the actual values of υ_L and υ_H. In particular, if (p_L^*, p_H^*, λ^*) is a solution to (8) given some υ_L and υ_H , then it is also a solution to (8) for any υ_L' and υ_H' such that $\upsilon_H' - \upsilon_L' = \upsilon_H - \upsilon_L$. Consider the case when υ_H and υ_L are such that $p_L^* > \upsilon_L$ and (p_L^*, p_H^*, λ^*) violates participation constraints given υ_L and υ_H. Then, triplet (p_L^*, p_H^*, λ^*) characterizes an equilibrium with negative information without violating the participating constraints for any values $\upsilon_H' > \upsilon_L' > p_L^*$, such that $\upsilon_H' - \upsilon_L' = \upsilon_H - \upsilon_L$.

[9] For the sake of simplicity, we assume that indifferent buyers always purchase the product.

References

Atiyeh, Clifford. Everything you need to know about the VW diesel-emissions scandal. Car and Driver, 2016. (https://web.archive.org/web/20200705083431/https://www.caranddriver.com/news/a15339250/everything-you-need-to-know-about-the-vw-diesel-emissions-scandal/)

Bauer, Raymond A. Consumer behavior as risk taking. In Hancock RS (ed) *Dynamic Marketing for a Changing World.* Chicago: American Marketing Association, 1960: 389-398

Berger, Jonah, Sorensen, Alan T., and Rasmussen, Scott J. "Positive effects of negative publicity: When negative reviews increase sales." Marketing science 29 (No. 5 2010): 815-827.

Board, Oliver. "Competition and disclosure." *The Journal of Industrial Economics* 57 (No. 2009): 197-213.

Busch, Paul, and Wilson, David T. "An experimental analysis of a salesman's expert and referent bases of social power in the buyer-seller dyad." *Journal of marketing research* 13 (No. 1 1976): 3-11.

Cheong, Insuk, and Kim, Jeong-Yoo. "Costly information disclosure in oligopoly." *The Journal of Industrial Economics* 52 (No. 1 2004): 121-132.

Chu, Junhong, and Chintagunta, Pradeep K. "Quantifying the economic value of warranties in the US server market." *Marketing Science* 28 (No. 1 2009): 99-121.

Crosby, Lawrence A., Evans, Kenneth R., and Cowles, Deborah. "Relationship quality in services selling: an interpersonal influence perspective." Journal of marketing 54 (No. 3 1990): 68-81.

Crowley, Ayn E., and Hoyer, Wayne D. "An integrative framework for understanding two-sided persuasion." *Journal of Consumer research* 20 (No. 4 1994): 561-574.

Dowling, Grahame R. "Perceived risk: the concept and its measurement." *Psychology & Marketing* 3 (No. 3 1986): 193-210.

Dranove, David, and Jin, Ginger Z. "Quality disclosure and certification: Theory and practice." Journal of Economic Literature 48 (No. 4 2010): 935-63.

Eisend, Martin. "Two-sided advertising: A meta-analysis." *International Journal of Research in Marketing* 23 (No. 2 2006): 187-198.

______. "Understanding two-sided persuasion: An empirical assessment of theoretical approaches." *Psychology & Marketing* 24 (No. 7 2007): 615-640.

Erdem, Tülin, Zhao, Ying, and, Valenzuela, Ana. "Performance of store brands: A cross-country analysis of consumer store-brand preferences, perceptions, and risk." *Journal of Marketing Research* 41 (No. 1 2004): 86-100.

Farrell, Joseph von Rosthorn. *Prices as signals of quality*. University of Oxford, *1981.*

Fay, Scott, and Xie, Jinhong. "The economics of buyer uncertainty: Advance selling vs. probabilistic selling." *Marketing Science* 29 (No. 6 2010): 1040-1057.

Gardete, Pedro M. "Cheap-talk advertising and misrepresentation in vertically differentiated markets." *Marketing Science* 32 (No. 4 2013): 609-621.

Geyskens, Inge, Steenkamp, Jan-Benedict EM, and Kumar, Nirmalya. "Generalizations about trust in marketing channel relationships using meta-analysis." *International Journal of Research in marketing* 15 (No. 3 1998): 223-248.

Grossman, Sanford J. "The informational role of warranties and private disclosure about product quality." *The Journal of Law and Economics* 24 (No. 3 1981): 461-483.

Grossman, Sanford J., and Hart, Oliver D. "Disclosure laws and takeover bids." *The Journal of Finance* 35 (No. 2 1980): 323-334.

Guo, Liang, and Zhao, Ying. "Voluntary quality disclosure and market interaction." *Marketing Science* 28 (No. 3 2009): 488-501.

Heal, Geoffrey. "Do bad products drive out good?" *The Quarterly Journal of Economics* 90 (No. 3 1976): 499-502.

Hotz, V. Joseph, and Xiao, Mo. "Strategic information disclosure: The case of multiattribute products with heterogeneous consumers." *Economic Inquiry* 51 (No. 1 2013): 865-881.

Kalra, Ajay, and Shi, Mengze. "Designing optimal sales contests: A theoretical perspective." *Marketing Science* 20 (No. 2 2001): 170-193.

Janssen, Maarten CW, and Teteryatnikova, Mariya. "Horizontal product differentiation: Disclosure and competition." *The Journal of Industrial Economics* 64 (No. 4 2016): 589-620.

Kim, Kyungmin. "Endogenous market segmentation for lemons." *The RAND Journal of Economics* 43 (No. 3 2012): 562-576.

Kourouxous, Thomas, and Bauer, Thomas. “Violations of dominance in decision-making.” *Business Research* 12 (No. 1 2019): 209-239.

Lichtenstein, Donald R., and Burton, Scot. “The relationship between perceived and objective price-quality.” *Journal of marketing research* 26 (No. 4 1989): 429-443.

Markin, Rom J. *Consumer Behavior: A Cognitive Orientation*. New York: Macmillan, 1974.

Masatlioglu, Yusufcan, and Raymond, Collin. “A behavioral analysis of stochastic reference dependence.” *American Economic Review* 106 (No. 9 2016): 2760-82.

Milgrom, Paul R. “Good news and bad news: Representation theorems and applications.” *The Bell Journal of Economics* 12 (No. 2 1981): 380-391.

Rao, Akshay R., and Monroe, Kent B. “The effect of price, brand name, and store name on buyers’ perceptions of product quality: An integrative review.” *Journal of marketing Research* 26 (No. 3 1989): 351-357.

Riordan, Michael H. “Monopolistic competition with experience goods.” *The Quarterly Journal of Economics* 101 (No. 2 1986): 265-279.

Ross, Ivan. “Perceived risk and consumer behavior: a critical review.” Advances in Consumer Research 2 (No. 1 1975):1-20.

Rubel, Olivier, and Prasad, Ashutosh. “Dynamic incentives in sales force compensation.” *Marketing Science* 35 (No. 4 2016): 676-689.

Ryu, Suyeol, and Kim, Iltae. “Conjectures in cournot duopoly under cost uncertainty.” *Seoul Journal of Economics* 24 (No. 1 2011): 73-86.

Settle, Robert B., and Golden, Linda L. “Attribution theory and advertiser credibility.” *Journal of Marketing Research* 11 (No. 2 1974): 181-185.

Shaked, Avner, and Sutton, John. “Relaxing price competition through product differentiation.” *The review of economic studies* 49 (No. 1 1982): 3-13.

Shapiro, Carl. “Premiums for high quality products as returns to reputations.” *The quarterly journal of economics* 98 (No. 4 1983): 659-679.

Shapiro, Dmitry. “Profitability of the name-your-own-price channel in the case of risk-averse buyers.” *Marketing Science* 30 (No. 2 2011): 290-304.

Shapiro, Dmitry, and Huh, Seung. “Incentives of low-quality sellers to disclose negative information.” *Journal of Economics &*

Management Strategy 30 (No. 1 2021): 81-99.

Smallwood, Dennis E., and Conlisk, John. "Product quality in markets where consumers are imperfectly informed." *The Quarterly Journal of Economics* 93 (No. 1 1979): 1-23.

Spulber, Daniel F. "Bertrand competition when rivals' costs are unknown." *The Journal of Industrial Economics* 43 (No. 1 1995): 1-11.

Stone, Robert N., and Winter, Frederick. *Risk in buyer behavior contexts: A clarification.* College of Commerce and Business Administration. University of Illinois at Urbana-Champaign, 1985.

Taylor, James W. "The role of risk in consumer behavior: A comprehensive and operational theory of risk taking in consumer behavior." *Journal of marketing* 38 (No. 2 1974): 54-60.

Viscusi, W. Kip. "A note on" lemons" markets with quality certification." *The Bell Jou*rnal of Economics 9 (No. 1 1978): 277-279.

Völckner, Franziska, and Hofmann, Julian. "The price-perceived quality relationship: A meta-analytic review and assessment of its determinants." Marketing letters 18 (No. 3 2007): 181-196.

Wilson, Robert. "Reputations in games and markets." Game-theoretic models of bargaining 96 (1985): 27-62.